

1 Exercises

1. Let $f(x, y) = x + 2y$. Evaluate the scalar line integral $\int_{\mathbf{x}} f \, ds$ over the given path \mathbf{x} .

(a) $\mathbf{x}(t) = (2 - 3t, 4t - 1), 0 \leq t \leq 2$

(b) $\mathbf{x}(t) = (\cos t, \sin t), 0 \leq t \leq \pi$

In Exercises 2–7, calculate $\int_{\mathbf{x}} f \, ds$, where f and \mathbf{x} are as indicated.

2. $f(x, y, z) = xyz, \mathbf{x}(t) = (t, 2t, 3t), 0 \leq t \leq 2$

3. $f(x, y, z) = \frac{x+z}{y+z}, \mathbf{x}(t) = (t, t, t^{3/2}), 1 \leq t \leq 3$

4. $f(x, y, z) = 3x + xy + z^3, \mathbf{x}(t) = (\cos 4t, \sin 4t, 3t), 0 \leq t \leq 2\pi$

5. $f(x, y, z) = \frac{z}{x^2 + y^2}, \mathbf{x}(t) = (e^{2t} \cos 3t, e^{2t} \sin 3t, e^{2t}), 0 \leq t \leq 5$

6. $f(x, y, z) = x + y + z,$

$$\mathbf{x}(t) = \begin{cases} (2t, 0, 0) & \text{if } 0 \leq t \leq 1 \\ (2, 3t - 3, 0) & \text{if } 1 \leq t \leq 2 \\ (2, 3, 2t - 4) & \text{if } 2 \leq t \leq 3 \end{cases}$$

7. $f(x, y, z) = 2x - y^{1/2} + 2z^2,$

$$\mathbf{x}(t) = \begin{cases} (t, t^2, 0) & \text{if } 0 \leq t \leq 1 \\ (1, 1, t - 1) & \text{if } 1 \leq t \leq 3 \end{cases}$$

In Exercises 8–16, find $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$, where the vector field \mathbf{F} and the path \mathbf{x} are given.

8. $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \mathbf{x}(t) = (2t + 1, t, 3t - 1), 0 \leq t \leq 1$

9. $\mathbf{F} = (y + 2)\mathbf{i} + x\mathbf{j}, \mathbf{x}(t) = (\sin t, -\cos t), 0 \leq t \leq \pi/2$

10. $\mathbf{F} = x\mathbf{i} + y\mathbf{j}, \mathbf{x}(t) = (2t + 1, t + 2), 0 \leq t \leq 1$

11. $\mathbf{F} = (y - x)\mathbf{i} + x^4 y^3 \mathbf{j}, \mathbf{x}(t) = (t^2, t^3), -1 \leq t \leq 1$

12. $\mathbf{F} = x\mathbf{i} + xy\mathbf{j} + xyz\mathbf{k}, \mathbf{x}(t) = (3 \cos t, 2 \sin t, 5t), 0 \leq t \leq 2\pi$

13. $\mathbf{F} = -3y\mathbf{i} + x\mathbf{j} + 3z^2\mathbf{k}, \mathbf{x}(t) = (2t + 1, t^2 + t, e^t), 0 \leq t \leq 1$

14. $\mathbf{F} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}, \mathbf{x}(t) = (t, 3t^2, 2t^3), -1 \leq t \leq 1$

15. $\mathbf{F} = 3z\mathbf{i} + y^2\mathbf{j} + 6z\mathbf{k}, \mathbf{x}(t) = (\cos t, \sin t, t/3), 0 \leq t \leq 4\pi$

16. $\mathbf{F} = y \cos z \mathbf{i} + x \sin z \mathbf{j} + xy \sin z^2 \mathbf{k}, \mathbf{x}(t) = (t, t^2, t^3), 0 \leq t \leq 1$

17. Determine the value of $\int_{\mathbf{x}} x \, dy - y \, dx$, where $\mathbf{x}(t) = (\cos 3t, \sin 3t), 0 \leq t \leq \pi$.