

Week in Review # 10
Sections 10.1, 10.2(some)

1. A cup of coffee contains about 250 mg of caffeine. Caffeine is metabolized and leaves the body at a continuous rate of 21% every hour. Write a differential equation that measures the amount of caffeine, A , in the body as a function of the number of hours, x , since the coffee was consumed.

A = # of mg. of caffeine

Rate = Rate in - Rate out

A' = rate in mg/hr

$$A' = 0 - .21A$$

$$A' = -.21A$$

2. Dead leaves accumulate on the ground in a forest at a rate of 4 grams per square centimeter per year. At the same time, these leaves decompose at a continuous rate of 70% per year. Write a differential equation for the total quantity of dead leaves (per square centimeter) at time x .

L = amt of leaves in grams per sq. cm.

L' = Rate grams per sq. cm / yr.

$$L' = 4 - .7L$$

3. A patient is given a drug intravenously at a rate of 43.2 mg/hour. The rate at which the drug leaves the body is proportional to the quantity present. When there are 100mg of the drug in a patient, then the rate that the drug is leaving is 8.2 mg/hour. Write a differential equation that measures the amount of the drug in the body as a function of the number of hours since it was started.

$A =$ amt of drug in mg

$A' =$ rate in mg/hr.

$$A' = \begin{array}{l} \text{rate in} \\ 43.2 \end{array} - \begin{array}{l} \text{Rate out.} \\ .082 A \end{array}$$

$$\text{Rate out} = k A$$

$$8.2 = k(100)$$

$$.082 = k$$

$$\text{Rate out} = .082 A$$

4. A population of fish was modeled by the following differential equation where P has units of millions of fish, and P' has units of millions of fish per year.

$$P' = .45P - 18$$

Assuming the the population started at 85 million fish and that time is measured from this starting value.

- (a) Use the information provided to approximate the population of the fish at the indicated values of t .

t	P (millions)
0	85
1	105.25
2	134.6125
3	177.1881

$$P' = .45(85) - 18$$

$$= 20.25 \text{ mill/yr}$$

$$\text{New pop} = 85 + 1(20.25)$$

$$= 105.25 \text{ million}$$

$$P' = .45(105.25) - 18$$

$$= 29.3625$$

$$\text{New pop.} = 105.25 + 1(29.3625)$$

- (b) Use the information provided to approximate the population of the fish at the indicated values of t .

t	P (millions)
0	85
0.5	95.125
1	107.5281
1.5	122.7219
2	141.3343
2.5	164.1345
3	192.0648

$$P' = .45P - 18$$

$$P' = .45(85) - 18 \\ = 20.25$$

$$\text{new population} = 85 + (.5)(20.25) \\ = 95.125$$

Repeat the method

(c) Show that $P = 40 + Ce^{0.45t}$ is a solution to the differential equation.

$$P' = .45Ce^{.45t}$$

$$P' = .45P - 18$$

Left side,

$$.45Ce^{.45t}$$

↑
equal

Right side

$$.45(40 + Ce^{.45t}) - 18$$

$$.45(40) + .45Ce^{.45t} - 18$$

$$18 + .45Ce^{.45t} - 18$$

$$.45Ce^{.45t}$$

So it is a solution.

(d) Find the value of C in part (c)

The initial population (at the start i.e. $t=0$) is 85 million.

$$85 = 40 + Ce^0$$

$$85 = 40 + C$$

$$C = 45$$

(e) Which method, part (a) or part (b), gave a better approximation to the population of fish three years after the start?

$$P = 40 + 45e^{.45t}$$

$$\text{at } t = 3$$

$$P = 40 + 45e^{.45(3)}$$

$$= 213.5841 \text{ million fish}$$

part (b) gave a better estimate.

5. Is $y = x^3 + 2x + 7$ is a solution to the differential equation $3y - xy' = 4x + 18$?

$$y' = 3x^2 + 2$$

Left side

Right side

$$3(x^3 + 2x + 7) - x(3x^2 + 2)$$

$$3x^3 + 6x + 21 - 3x^3 - 2x$$

$$4x + 21$$

$$4x + 18$$

Not equal so it is not
a solution.

6. Is $y = 2e^{5x} + 3x$ a solution to the differential equation $y'' - 4y' + 12 = 5y - 15x$?

$$y' = 10e^{5x} + 3$$

$$y'' = 50e^{5x}$$

L.S.

$$50e^{5x} - 4(10e^{5x} + 3) + 12$$

$$50e^{5x} - 40e^{5x} - 12 + 12$$

$$10e^{5x}$$

R.S.

$$5(2e^{5x} + 3x) - 15x$$

$$10e^{5x} + 15x - 15x$$

$$10e^{5x}$$

equal so it is a solution.

7. Find the value of k so that $y = x^4 + kx$ is a solution to the differential equation $4y - xy' = 30x$

$$y' = 4x^3 + k$$

$$4(x^4 + kx) - x(4x^3 + k) = 30x$$

$$\cancel{4x^4} + 4kx - \cancel{4x^4} - kx = 30x$$

$$3kx = 30x$$

this implies that

$$3k = 30$$

or $k = 10$

8. Find the values of c and k such that $y = ce^{kx}$ is a solution to the differential equation $5y' = 3y$.

$$y' = kce^{kx}$$

$$5(kce^{kx}) = 3(ce^{kx})$$

$$5kc e^{kx} = 3ce^{kx}$$

$$5kc = 3c$$

Since c is on both sides & is multiplied

Then c can be any #

for this to work we need

$$5k = 3$$

or $k = \frac{3}{5}$

since e^{kx} is
never zero
divide both
sides by e^{kx}