Week in Review \# 10
Sections 10.1, 10.2 (some)

## Things to know:

- Be able to setup a differential equation.
- Understand what it means to be a solution to a differential equation.

1. A cup of coffee contains about 250 mg of caffeine. Caffeine is metabolized and leaves the body at a continuous rate of $21 \%$ every hour. Write a differential equation that measures the amount of caffeine, $A$, in the body as a function of the number of hours, $x$, since the coffee was consumed.
2. Dead leaves accumulate on the ground in a forest at a rate of 4 grams per square centimeter per year. At the same time, these leaves decompose at a continuous rate of $70 \%$ per year. Write a differential equation for the total quantity of dead leaves( per square centimeter) at time $x$.
3. A patient is given a drug intravenously at a rate of $43.2 \mathrm{mg} /$ hour. The rate at which the drug leaves the body is proportional to the quantity present. When there are 100 mg of the drug in a patient, then the rate that the drug is leaving is $8.2 \mathrm{mg} / \mathrm{hour}$. Write a differential equation that measures the amount of the drug in the body as a function of the number of hours since it was started.
4. A population of fish was modeled by the following differential equation where $P$ has units of millions of fish, and $P^{\prime}$ has units of millions of fish per year.

$$
P^{\prime}=.45 P-18
$$

Assuming the the population started at 85 million fish and that time is measured from this starting value.
(a) Use the information provided to approximate the population of the fish at the indicated values of $t$.

| $t$ | $P$ |
| :---: | :---: |
| 0 | 85 |
| 1 |  |
| 2 |  |
| 3 |  |

(b) Use the information provided to approximate the population of the fish at the indicated values of $t$.

| $t$ | $P$ |
| :---: | :---: |
| 0 | 85 |
| 0.5 |  |
| 1 |  |
| 1.5 |  |
| 2 |  |
| 2.5 |  |
| 3 |  |

(c) Show that $P=40+C e^{0.45 t}$ is a solution to the differential equation.
(d) Find the value of C in part (c)
(e) Which method, part (a) or part (b), gave a better approximation to the population of fish three years after the start?
5. Is $y=x^{3}+2 x+7$ is a solution to the differential equation $3 y-x y^{\prime}=4 x+18$ ?
6. Is $y=2 e^{5 x}+3 x$ a solution to the differential equation $y^{\prime \prime}-4 y^{\prime}+12=5 y-15 x$ ?
7. Find the value of $k$ so that $y=x^{4}+k x$ is a solution to the differential equation $4 y-x y^{\prime}=30 x$
8. Find the values of $c$ and $k$ such that $y=c e^{k x}$ is a solution to the differential equation $5 y^{\prime}=3 y$.

