Chapter 9 Homework Solutions
Compiled by Joe Kahlig

Note in problems 1–6 the columns are labeled in alphabetical order from the left to right.

1. $T = \begin{bmatrix} 0.25 & 0.4 \\ 0.75 & 0.6 \end{bmatrix}$

2. $T = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}$

3. $T = \begin{bmatrix} 0.1 & 0.7 & 0.5 \\ 0.8 & 0.2 & 0.4 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$

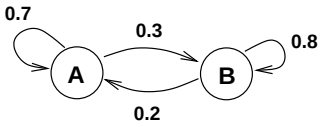
4. $T = \begin{bmatrix} 0.2 & 0.8 & 0.6 \\ 0.5 & 0.1 & 0 \\ 0.3 & 0.1 & 0.4 \end{bmatrix}$

5. $T = \begin{bmatrix} 0.4 & 0 & 0.3 \\ 0 & 0.2 & 0.7 \\ 0.6 & 0.8 & 0 \end{bmatrix}$

6. $T = \begin{bmatrix} 1 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.7 & 0 \\ 0 & 0.6 & 0 & 0 & 0.8 \\ 0 & 0 & 0 & 0.3 & 0.2 \\ 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$

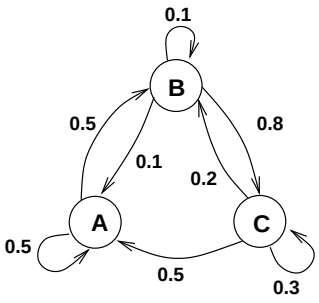
7. no, columns don't add to 1

8. yes

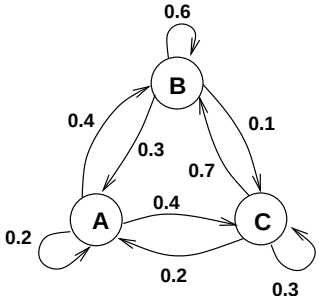


9. no, not a square matrix.

10. yes



11. yes



12. no, last column doesn't add to 1.

13. (a) $T_{1,2} = 0.2$ means that knowing that you are in state B, the probability of moving to state A is 20%

    (b) 0.6

    (c) $X_1 = TX_0 = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$

    26% in state A
    74% in state B

    (d) $T^3_{2,2} = .752$ means that 75.2% of those starting in state B will still be in state B after three iterations of the Markov process.

    (e) $T^3_{2,1} = .744$
    Answer: 74.4% of those who start off in state A will be in state B.

14. (a) $T_{2,3} = 0.4$ means that knowing that you are in state C, the probability of moving to state B is 40%

    (b) 0.15

    (c) 0.2

    (d) $X_1 = TX_0 = \begin{bmatrix} 0.375 \\ 0.43 \\ 0.195 \end{bmatrix}$

    37.5% in state A
    43% in state B
    19.5% in state C

    (e) $T^4_{2,3} = .3572$ means that 35.72% of those starting in state C will be in state B after four iterations of the Markov process.

    (f) $T^4_{1,2} = 0.39755$
    Answer: 39.755% of those who start off in state B will be in state A.

15. $X_3 = T^3X_0 = \begin{bmatrix} 0.2575 \\ 0.56534 \\ 0.17716 \end{bmatrix}$

16. $X_4 = T^4X_0 = \begin{bmatrix} 0.42048 \\ 0.3783 \\ 0.20122 \end{bmatrix}$

17. $X_2 = TX_1 = \begin{bmatrix} 0.31 \\ 0.31 \\ 0.38 \end{bmatrix}$

18. $X_4 = T^2X_2 = \begin{bmatrix} 0.242 \\ 0.448 \\ 0.31 \end{bmatrix}$

19. $X_6 = T^3X_3 = \begin{bmatrix} 0.3524 \\ 0.2941 \\ 0.3534 \end{bmatrix}$

20. $X_4 = T^4X_0 = \left(T^2\right)^2 X_0 = \begin{bmatrix} 0.3585 \\ 0.2898 \\ 0.3518 \end{bmatrix}$

21. (a) State B = business major
State O = other major

$$\text{T} = \begin{array}{c} \\ \text{State B} \\ \text{State O} \end{array} \begin{array}{c} \begin{array}{cc} \text{State} & \\ \text{B} & \text{O} \end{array} \\ \begin{bmatrix} 0.88 & 0.25 \\ 0.12 & 0.75 \end{bmatrix} \end{array}$$

(b) $X_6 = T^6X_0 = \begin{bmatrix} 0.658439 \\ 0.3415605 \end{bmatrix}$

Answer: 65.8439%

(c) $T^6_{1,1} = 0.695954$

Answer: 69.5954%

22. (a) State R = rural population
State U = urban population

$$\text{T} = \begin{array}{c} \\ \text{State R} \\ \text{State U} \end{array} \begin{array}{c} \begin{array}{cc} \text{State} & \\ \text{R} & \text{U} \end{array} \\ \begin{bmatrix} 0.96 & 0.02 \\ 0.04 & 0.98 \end{bmatrix} \end{array}$$

(b) $T^3_{2,1} = 0.112944$

Answer: 11.2944%

(c) 53.8211% rural
46.1789% urban

(d) 43.0839% rural
56.9161% urban

23. (a) State U = the University Bookstore
State T = Textbooks for Less
State A = A-plus Books

$$\text{T} = \begin{array}{c} \\ \text{State U} \\ \text{State T} \\ \text{State A} \end{array} \begin{array}{c} \begin{array}{ccc} \text{State} & \text{State} & \text{State} \\ \text{U} & \text{T} & \text{A} \end{array} \\ \begin{bmatrix} 0.8 & 0.05 & 0.05 \\ 0.1 & 0.7 & 0.20 \\ 0.1 & 0.25 & 0.75 \end{bmatrix} \end{array}$$

(b) $X_3 = T^3X_0 = \begin{bmatrix} 0.284375 \\ 0.32875 \\ 0.386875 \end{bmatrix}$

Answer: 32.875

(c) $X_6 = T^6X_0 = \begin{bmatrix} 0.235596 \\ 0.346074 \\ 0.418330 \end{bmatrix}$

Answer:
23.5596% for the University Bookstore
34.6074% for Textbooks for Less
41.8330% for A-plus Books

24. (a) State A = brand A
State B = brand B
State C = brand C

$$\text{T} = \begin{array}{c} \\ \text{State A} \\ \text{State B} \\ \text{State C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{State} & \text{State} & \text{State} \\ \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0.97 & 0.02 & 0.04 \\ 0.02 & 0.95 & 0.03 \\ 0.01 & 0.03 & 0.93 \end{bmatrix} \end{array}$$

(b) $T^3_{1,2} = 0.058752$

Answer: 5.8752%

(c) $X_4 = T^4X_0 = \begin{bmatrix} 0.396902 \\ 0.415012 \\ 0.188086 \end{bmatrix}$

Answer: 39.6902% for brand A
41.5012% for brand B
19.8086% for brand C.

25. regular

26. not regular

27. not regular

28. regular

29. not regular

30. regular

31. $X = \begin{bmatrix} \frac{8}{23} \\ \frac{15}{23} \end{bmatrix}$

32. $X = \begin{bmatrix} .4 \\ .6 \end{bmatrix}$

33. $X = \begin{bmatrix} \frac{17}{37} \\ \frac{18}{37} \\ \frac{2}{37} \end{bmatrix}$

34. $X = \begin{bmatrix} \frac{29}{73} \\ \frac{26}{73} \\ \frac{18}{73} \end{bmatrix}$

35. 33.333% will live in rural areas.

36. 20% for the University Bookstore
36% for Textbooks for Less
44% for A-plus Books

37. 48.1481% for product A
31.4815% for product B
20.3704% for product C

38. none of the states are absorbing

39. B

40. A

41. B

42. A and C

43. none

44. A

45. no, it is not an absorbing Markov process

46. yes, T =

	A	B	C	D	E
A	1	0.4	0	0	0
B	0	0	0.5	0.7	0
C	0	0.6	0	0	0.8
D	0	0	0	0.3	0.2
E	0	0	0.5	0	0

47. yes, T =

	C	D	A	B
C	1	0	0.35	0.5
D	0	1	0.3	0.15
A	0	0	0.35	0.15
B	0	0	0	0.2

48. yes, T =

	A	C	B	D
A	1	0	0.2	0
C	0	1	0	0.5
B	0	0	0.7	0
D	0	0	0.1	0.5

49. no, it is not an absorbing Markov process

50. no, it is not an absorbing Markov process

51. yes, T =

	B	A	C
B	1	0.8	0.5
A	0	0.1	0.4
C	0	0.1	0.1

52. yes, T =

	B	C	A	D
B	1	0	0.2	0.3
C	0	1	0.4	0.3
A	0	0	0.1	0.2
D	0	0	0.3	0.2

53. yes,

	C	A	B	D
C	1	0	0.3	0.4
A	0	0	0.7	0
B	0	0	0	0.6
D	0	1	0	0

54. L=

	A	B	C	D
A	1	0	0.379	0.470
B	0	1	0.621	0.530
C	0	0	0	0
D	0	0	0	0

F =

	C	D
C	1.212	0.303
D	0.455	1.264

55. L=

	A	B	C	D
A	1	0	0	0.286
B	0	1	0	0.143
C	0	0	1	0.571
D	0	0	0	0

F =

	D
D	1.429

56. L =

	A	B	C
A	1	0	0.6
B	0	1	0.4
C	0	0	0

F =

	C
C	2

57. L=

	A	B	C	D	E
A	1	0	0.465	0.692	0.434
B	0	1	0.535	0.308	0.566
C	0	0	0	0	0
D	0	0	0	0	0
E	0	0	0	0	0

F =

	C	D	E
C	1.938	0.769	1.015
D	0.677	1.538	0.831
E	0.738	0.769	1.815

58. L=

	A	B	C	D	E
A	1	0	0	0.289	0.444
B	0	1	0	0.6	0
C	0	0	1	0.111	0.556
D	0	0	0	0	0
E	0	0	0	0	0

F =

	D	E
D	2	0
E	0.444	2.222

59. A: 28.6%
B: 14.3%
C: 57.1%

60. (a) A: 37.9%
B: 62.1%

(b) A: 47.0%
B: 53.0%

(c) C: $1.212 + 0.455 = 1.667$ iterations
D: $0.303 + 1.264 = 1.567$ iterations

(d) 1.212 times

(e) 0.455 times

61. (a) 46.5%

(b) 56.6%

(c) D

(d) E

(e) 1.938 times

(f) 0.769 times

(g) D

(h) E

62. Find the Limiting and the fundamental matrices.

$$L= \begin{array}{c} \\ \text{E-1} \\ \text{E-2} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array} \begin{bmatrix} \text{E-1} & \text{E-2} & \text{A} & \text{B} & \text{C} \\ 1 & 0 & 0.526 & 0.421 & 0.579 \\ 0 & 1 & 0.484 & 0.579 & 0.421 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{bmatrix} \text{A} & \text{B} & \text{C} \\ 2.53 & 1.42 & 1.58 \\ 1.89 & 2.32 & 1.68 \\ 2.63 & 2.11 & 2.89 \end{bmatrix}$$

(a) 52.6%

(b) 42.1%

(c) A

(d) $2 * 7.05 = 14.1$ minutes

(e) $2.63 * 2 = 5.26$ minutes

(f) $2 * 1.42 = 2.84$ minutes

(g) 5.85

63. Find the Limiting and the fundamental matrices.

$$L= \begin{array}{c} \\ \text{die} \\ \text{home} \\ \text{ICU} \\ \text{CCW} \end{array} \begin{bmatrix} \text{die} & \text{home} & \text{ICU} & \text{CCW} \\ 1 & 0 & 0.146 & 0.087 \\ 0 & 1 & 0.854 & 0.913 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{array}{c} \\ \text{ICU} \\ \text{CCW} \end{array} \begin{bmatrix} \text{ICU} & \text{CCW} \\ 1.943 & 0.353 \\ 3.416 & 3.651 \end{bmatrix}$$

(a) 85.4%

(b) 8.7%

(c) 5.4 days