Chapter 4 Homework problems
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Section 4.1

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1. Setup the initial simplex matrix.

   (a) Maximize: $f = -x + 2y + z$
   Constraints:
   $y + 2z \leq 40$
   $x + y + 2z \leq 60$
   $x + z \leq 30$
   $x, y, z \geq 0$

   (b) Maximize: $P = 3x + 6y - 5z$
   Constraints:
   $2x + y + z \leq 14$
   $4x + 2y + 3z \leq 28$
   $z \leq 10$
   $x \geq 0, y \geq 0, z \geq 0$

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2. Determine if the simplex method has been completed in these tableaus. If it is finished, then read off the solution. If it is not finished, then find the next pivot element, if it exists, by the methods shown in class.

(a)

x	y	s_1	s_2	f	
4	2	1	0	0	4
-3	6	0	1	0	18
-2	-4	0	0	1	0

(b)

x	y	z	s_1	s_2	f	
0	1	1	-1	-1	0	2
1	-1	0	2	1	0	4
0	1	0	1	3	1	1

(c)

x	y	s_1	s_2	f	
1	-2	1	0	0	8
3	-4	0	1	0	12
-1	-4	0	0	1	0

(d)

x	y	z	s_1	s_2	s_3	f	
3	-2	0	2	0	1	0	4
2	3	1	-2	0	0	0	60
1	4	0	8	1	0	0	20
4	-4	0	-1	0	0	1	10

(e)

x	y	z	s_1	s_2	s_3	f	
0	3	5	0	4	1	0	10
1	2	6	0	7	0	0	5
0	0	3	1	-2	0	0	3
0	-3	-5	0	6	0	1	18

(f)

x	y	z	s_1	s_2	s_3	P	
0	1	-4	3	0	5	0	5
0	0	2	6	1	-2	0	7
1	0	3	9	0	4	0	6
0	0	9	6	0	12	1	29

(g)

x	y	z	s_1	s_2	s_3	P	
3	-2	0	2	0	1	0	4
2	3	1	-2	0	0	0	0
1	4	0	8	1	0	0	20
4	-1	0	-4	0	0	1	10

(h)

x	y	z	s_1	s_2	s_3	P	
6	0	-2	0	2	1	0	12
2	1	6	0	-4	0	0	10
3	0	2	1	3	0	0	4
-2	0	3	0	-1	0	1	16

(i)

x	y	z	w	s_1	s_2	s_3	s_4	P	
3	5	6	1	1	0	0	0	0	9
2	8	2	0	3	1	0	0	0	6
5	4	3	0	-4	0	1	0	0	0
7	2	1	0	0	0	0	1	0	15
-4	5	12	0	-5	0	0	0	1	140

3. The following is the last simplex matrix for a problem that a student was working last night.
   A) Read off the solution for all of the variables.
   B) Would the answer from part (A) be a correct solution to the linear programming problem? If this can not be decided, then tell what additional information will be needed.

x	y	z	s_1	s_2	s_3	P	
5	0	0	1	-1	1	0	-9
2	1	0	0	3	5	0	5
3	0	1	0	-4	2	0	3
7	0	0	0	2	5	1	40

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4. Solve these linear programming problems using the simplex method.

   (a) Maximize: $f = -x + 2y + z$
   Constraints:
   $y + 0.4z \leq 40$
   $x + y + 2z \leq 60$
   $x + z \leq 50$
   $x, y, z \geq 0$

   (b) Maximize: $P = 23x + 24y + 16z$
   Constraints:
   $3x + y + 2z \leq 119$
   $2x + 2y + z \leq 97$
   $x + 2y + 3z \leq 136$
   $x \geq 0, y \geq 0, z \geq 0$

   (c) Maximize: $P = 6x - 4y + 2z$
   Constraints:
   $2x + y + 2z \leq 10$
   $4x - 3y + z \leq 20$
   $y \leq 10$
   $x \geq 0$
   $y \geq 0$
   $z \geq 0$

Solve these simplex problems and answer the additional questions.

5. A confectioner makes three types of candy: Sweet Tooth, Sugar Dandy, and Dandy Delite. Each box of candy has the following requirements. The current inventory of each ingredient is also listed in the table.

	Sweet Tooth	Sugar Dandy	Dandy Delite	inventory
chocolate	3 lbs	4 lbs	5 lbs	725 lbs
nuts	1 lb	0.5 lbs	0.75 lbs	130 lbs
fruit	1 lb		1 lb	70 lbs

Each box of Sweet Tooth sells for $8, Sugar Dandy sells for $5, and Dandy Delite sells for $6.

(a) How many boxes of <u>each type</u> of candy should be made from the available inventory in order to maximize the revenue?

(b) After maximizing the revenue, is there any inventory left over? If yes, then list the left over ingredient(s) and their amounts.

(c) Use shadow values to determine how much revenue will the company make if its supply of chocolate increases to 730 lbs and its supply of nuts increased to 140 lbs.

6. Your umbrella company makes three models: the Sprinkle, the Storm, and the Hurricane. The amounts of cloth, metal, and wood used in making each model are given in the table.

	Sprinkle	Storm	Hurricane	Available
Cloth (square yard)	1	2	2	300
metal (pounds)	2	1	3	800
wood (pounds)	1	3	6	600

The profit for the Sprinkle is $1, for the Storm is $1 and for the Hurricane is $2.

(a) How many of each type of umbrella should be produced so that profit is maximized?

(b) After maximizing the profit, is there any inventory left over? If yes, then list the left over resource(s) and their amounts.

(c) Use shadow values to determine the profit will the company make if its supply of cloth increases by 50 square yards and its supply of wood increased to 650 lbs.

(d) Are there any other valid solutions to this problem? if so find them.

7. A company makes three different types of desks. Let **x**, **y**, and **z** be the number of executive, office, and student desks made and sold each day, respectively. There are 16 cabinet hours, 18 finishing hours, and 10 crating hours available at the company each day. Each desk used various amount of cabinet making time, finishing time, and crating time. The linear programming problem that maximizes profit, while staying within the available resources is:

Maximize:
$P = 40x + 100y + 80z$

Constraints:
$2x + y + z \leq 16$ (Cabinet hours)
$x + 2y + 2z \leq 18$ (Finishing hours)
$x + y + .5z \leq 10$ (Crafting hours)
$x, y, z \geq 0$

(a) What should be produced in order to achieve maximum profit? List the quantities of each.

(b) Are there any resources left over when the maximum profit is achieved? If so, list all resources left over and the quantities of each.

(c) Use shadow values to determine how much profit the company will make if the company decreases its amount of finishing hours by 2 and its amount of crafting hours increases to 15.

8. A contractor is planning to build a new housing development. Let x, y, and z be the number of colonial, split-level, and ranch-style houses that are built in the housing development, respectively. The linear programming problem that maximizes profit, while staying within the available resources is:

Maximize:
P= 20,000x + 25,000y + 24,000z

Constraints:
$60,000x + 64,000y + 80,000z \leq 3,200,000$ dollars of capital
$4,000x + 3,000y + 4,000z \leq 180,000$ labor-hours available
$0.5x + 0.5y + z \leq 30$ acres of land
$x, y, z \geq 0$

(a) How many of each house, list quantities, should be built in order to achieve a maximum profit? (If no profit is possible, explain why.)

(b) Are there any resources left over when the maximum profit is achieved? If so, list all resources left over and the quantities of each.

(c) Are there any other valid solutions to this problem? if so find them.