Math 152 Week in Review: Sections 7.8(finish), 11.1, 11.2

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

1. Use the Comparison Theorem to determine if the integral converges or diverges

$$\int_2^\infty \frac{4x}{x^2 + e^{4x^2}}\, dx$$

2. Find a formula for the general term, a_n, of the sequence assuming that the pattern of the first few terms continues. Give the formula so the first term is a_1.

$$\left\{ \frac{1}{5}, \frac{-4}{9}, \frac{7}{13}, \frac{-10}{17}, \frac{13}{21}, \dots \right\}$$

3. Determine whether each sequence converges or diverges. If it converges, find the limit.

(a) $a_n = \dfrac{3n}{\ln(4n)}$

(b) $a_n = \dfrac{(-3)^n}{2^{2n}}$

(c) $a_n = \dfrac{(-1)^n n^2}{n^2+1}$

(d) $a_n = \sin(e^{-2n})$

(e) $a_n = \dfrac{\sin(3n)}{n+5}$

4. Determine whether the following sequence are increasing, decreasing, or not monotonic. Also determine if the sequence is bounded.

   (a) $a_n = 3 + \dfrac{1}{n}$

(b) $a_n = \ln(2+3n) - \ln(1+n)$

(c) $a_n = \cos\left(\frac{1}{n}\right)$

5. Assume that the sequence defined below is bounded and is decreasing. Determine if the sequence is convergent or divergent.

$$a_1 = 2, \qquad a_{n+1} = \frac{6}{7 - a_n}$$

6. Let $a_n = \dfrac{2n^2}{5n^2 - 3}$

(a) Determine whether $\{a_n\}$ is convergent.

(b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent

7. Assume the n-th partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \dfrac{3n+2}{1-2n}$.

(a) Find a_1 and a_4.

(b) Find a formula for a_n when $n > 1$

(c) Find $\sum_{n=1}^{\infty} a_n$.

8. Determine whether the following series are convergent or divergent. If a series is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{3e^{2n}}{e^{2n}+4}\right)$

(b) $\displaystyle\sum_{n=2}^{\infty} 5^{-n+1}3^{n}$

(c) $8 - 10 + \frac{25}{2} - \frac{125}{8} + \frac{625}{32} - \cdots$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n + 3^n}{5^n}$

9. Find the values of x for which the following series converge. Give your answer in interval notation. Find the sum of the series.

$$\sum_{n=1}^{\infty} 2^{n-1}(x-3)^n$$

10. Determine if this telescoping series converges or diverges. If it converges give the value.

$$\sum_{k=1}^{\infty} \left(e^{3/(k+2)} - e^{3/k}\right)$$