Math 152 Week in Review: Sections 11.8, 11.9

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

For a given power series centered at $x = a$, $\sum_{n=0}^{\infty} c_n(x-a)^n$, there are only three possibilities for convergence.

(i) The series converges only when $x = a$.

(ii) The series converges for all x.

(iii) The is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

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1. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 5$ but diverges when $x = -9$. What can be said about the convergence or divergence of the following series?

   A) $\sum_{n=0}^{\infty} c_n 10^n$ B) $\sum_{n=0}^{\infty} c_n$ C) $\sum_{n=0}^{\infty} (-1)^n c_n 5^n$ D) $\sum_{n=0}^{\infty} c_n(-2)^n$ E) $\sum_{n=0}^{\infty} c_n 9^n$

2. Find the interval and the radius of convergence for the following power series.

   (a) $\sum_{n=0}^{\infty} \frac{n5^n(x+3)^n}{3n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!(5x-1)^n}{n!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(2x-5)^n}{n4^n}$

Power Series Building Blocks:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad |x| < 1 \text{ so } R = 1 \text{ and } I : (-1, 1)$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} \qquad |x| < 1 \text{ so } R = 1 \text{ and } I : (-1, 1]$$

$$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad R = 1$$

3. Find a power series representation for the function. Determine the radius and interval of convergence.

$$g(x) = \frac{5x}{8 + 27x^3}$$

4. Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{x}{2x^6+3}\,dx$$

5. Find a power series representation for the function and the Radius of convergence.

$$f(x) = \frac{5}{(1-2x)^2}$$

6. Find a power series representation for the function and the Radius of convergence.

   $f(x) = \dfrac{3x^4}{(1+x)^3}$

7. Find a power series representation for the function.

$f(x) = \ln(1 + x^2)$

8. Find a power series representation for the function.

$f(x) = \ln(9 - x^3)$

9. Evaluate the indefinite integral as a power series.

$$\int x^2 \arctan(3x^2)\ dx$$

Be sure you learn these.

Important Maclaurin series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ... \qquad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ... \qquad R = \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \qquad R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... \qquad R = \infty$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - ... \qquad R = 1$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} = x - \frac{x^2}{2} + \frac{x^3}{3} - ... \qquad -1 < x \le 1$$