

Section 5.4 Homework Problems

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Section 5.4

1. A village has two industries, A and B. Production of one unit of A requires 0.3 units of A and 0.4 units of B. Production of one unit of B requires 0.2 units of A and 0.3 units of B.

- Find the input-output matrix, A .
- Find the production needed by the village if they want to export to the city, 4100 units of A and 3280 units of B.
- How much of this production is consumed internally?
- The village wants to increase the number of units of A exported by one. How does this effect the production?

2. A village has two industries: farming and woodcutting. They have found that to produce one unit of food it takes 0.3 units of food and 0.1 units of wood. To produce one unit of wood it take 0.5 units of food and 0.1 units of wood. All additional food and wood is exported to a nearby city . The city has a demand of 4350 units of food and 6728 units of wood.

- Give the input-output matrix.
- What level of production is needed by the village to meet the demand of the city?
- How much of the production is used internally by the village?
- Using the matrix $(I - A)^{-1}$, how many additional units of food and wood are needed to be produced if the demand for food increases by 1 unit?
- Using the matrix $(I - A)^{-1}$, how many additional units of food and wood are needed to be produced if the demand for wood increases by 3 units?

3. A village has three industries: farming, weaving, and pottery. They have found that to produce one unit of food it takes 0.3 units of food, 0.2 units of cloth and 0.1 units of pottery. To produce one unit of cloth it takes 0.2 units of food, 0.1 unit of cloth, and 0.2 units of pottery. To produce 1 unit of pottery it takes 0.2 units of food, 0.3 units of cloth, and 0.3 units of pottery.

- Give the input-output matrix.
- If the production level of the village is 40 units of food, 42 units of cloth and 28 units of pottery, how much can of the production will the village be able to export to a nearby city?
- What production level by the village is needed if the demand in the nearby city is 1356 units of cloth, 678 units of pottery, and 2034 units of food,?

(d) For a production level of 90 units of pottery, 150 units of cloth and 120 units of food, how much of the production is used by the village?

(e) Using the matrix $(I - A)^{-1}$, how many additional units of food, cloth and pottery are needed to be produced if the demand for cloth increases by 1 unit?

(f) Using the matrix $(I - A)^{-1}$, how many additional units of food, cloth and pottery are needed to be produced if the demand for pottery increases by 5 units?

4. An economy has three sectors: raw rubber (R), tires(T), and other rubber goods(G). The input-output matrix A is shown below including the sector labels.

$$A = \begin{bmatrix} & R & T & G \\ R & 0.08 & 0.6 & 0.3 \\ T & 0.04 & 0.02 & 0.01 \\ G & 0.02 & 0 & 0.06 \end{bmatrix}$$

Note: first row and column are not actually part of the matrix. They are the labels of the columns and rows.

(a) If there is demand for \$200 million for raw rubber, \$800 million for tires, and \$120 million for other rubber goods, find the production level of the economy that would satisfy this demand.

(b) How much of this production is consumed by internal production?

5. A village has found that to produce \$1 (1 unit) of food requires the usage of \$0.20 of food, \$0.10 of cloth, and \$0.10 of wood. To produce \$1 (1 unit) of cloth requires the usage of \$0.10 of food, \$0.10 of cloth, and \$0.30 of wood. To produce \$1 (1 unit) of wood requires the usage of \$0.10 of food, \$0.30 of cloth, and \$0.10 of wood. All of the additional food, cloth, and wood is exported to a nearby city to meet their demand.

(a) If the city has a demand of \$3200 worth of food, \$400 worth of cloth, and \$800 worth of wood, how much of each category should the village produce so that they meet its own needs and the city's needs?

(b) If the village produces \$5000 of food, \$2100 of cloth and \$3000 of wood, how much will they be able to export to the city? i.e. what demand could they meet for the city?

6. An economy is based on three sectors: coal, oil and transportation. Production of 1 unit of coal requires an input of 0.20 units from the coal sector and 0.40 units from the transportation sector. Production of 1 unit of oil requires an input of 0.1 units from the oil sector and 0.2 units from the transportation sector. Production of 1 unit of transportation requires an input of 0.4 units from the coal sector and 0.2 units from the oil sector and 0.2 units from the transportation sector.

- (a) Find the input-output matrix A .
- (b) Find the production for each sector that is needed to satisfy a final demand of \$40 billion for coal, \$25 billion for oil, and \$50 billion for transportation.
- (c) If the economy has production of \$120 billion for coal, \$60 billion for oil, and \$190 billion for transportation, then how much of this production is used in the production process of the economy (i.e. internally)? How much of this production is available for the demand of the consumers?

7. An economy has three sectors: R, T, and E. The input-output matrix A is shown below including the sector labels.

$$A = \begin{bmatrix} & \text{R} & \text{T} & \text{E} \\ \text{R} & 0.1 & 0.4 & 0.1 \\ \text{T} & 0.7 & 0.6 & 0.2 \\ \text{E} & 0.2 & 0 & 0.7 \end{bmatrix}$$

Note: first row and column are not actually part of the matrix. They are the labels of the columns and rows. What are the equilibrium production levels for these three sectors?

8. A village has found that to produce 1 unit of food requires the usage of 0.6 of food, 0.1 of cloth, and 0.3 of wood. To produce 1 unit of cloth requires the usage of 0.4 of food, 0.2 of cloth, and 0.4 of wood. To produce 1 unit of wood requires the usage of 0.6 of food, 0.2 of cloth, and 0.2 of wood. The village is closed to outsiders as such none of the production is available to be exported. What are the equilibrium production levels for these three sectors?
9. A village has found that to produce 1 unit of food requires the usage of 0.4 of food, 0.2 of cloth, and 0.4 of wood. To produce 1 unit of cloth requires the usage of 0.3 of food, 0.5 of cloth, and 0.2 of wood. To produce 1 unit of wood requires the usage of 0.6 of food, 0.2 of cloth, and 0.2 of wood. The village is closed to outsiders as such none of the production is available to be exported. What are the production levels for these three sectors when 7 unit of wood is produced?
10. A company makes 5 products: A, B, C, D, and E. Two of the products, A and B, are basic parts and do not require any of the other products in their production. To make 1 product C, requires 2 product A and 3 product B. To make one product D requires 4 product A, 2 product B and 1 product C. To make one product E requires 2 product A, 1 product C, and 3 product D.
- (a) If the company has a production level of 1000 product A, 815 product B, 170 product C, 115 product D and 25 product E, how much of the production will the company be able to sell?
 - (b) How much of the 815 product B produced were used to create the other products?

- (c) The company has an order for 6 product A, 9 product B, 22 product C, 15 product D, and 10 product E. What production level is needed to meet this order?