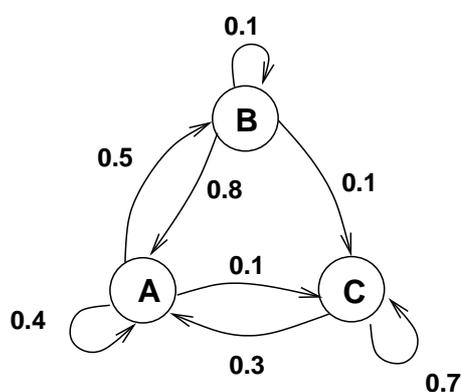


Week in Review # 9

Section M.1:

- Markov process
- Transition Diagram
- Transition Matrix/Stochastic Matrix
 - square matrix
 - entries are probability
 - columns sum to 1
- compute m^{th} state by $X_m = T^m X_o$

1. Give the transition matrix for the transition diagram.



2. Determine if the given matrix is a stochastic matrix. If it is, then draw the associated transition diagram.

(a)
$$\begin{matrix} A & \begin{bmatrix} A & B \\ 0.3 & 0.7 \\ 0.8 & 0.2 \end{bmatrix} \\ B & \end{matrix}$$

(b)
$$\begin{matrix} B & \begin{bmatrix} A & B \\ 0.4 & 0.7 \\ 0.6 & 0.3 \end{bmatrix} \\ A & \end{matrix}$$

(c)
$$\begin{matrix} A & \begin{bmatrix} A & B \\ .2 & .5 \\ .4 & .2 \end{bmatrix} \\ B & \\ C & \begin{bmatrix} .4 & .3 \end{bmatrix} \end{matrix}$$

(d)
$$\begin{matrix} A & \begin{bmatrix} A & B & C & D \\ 0.4 & 0.2 & 0 & 0.1 \\ 0.2 & 0 & 1 & 0.2 \\ 0.3 & 0.6 & 0 & 0.5 \\ 0.1 & 0.2 & 0 & 0.2 \end{bmatrix} \\ B & \\ C & \\ D & \end{matrix}$$

3. The transition matrix for a Markov process is given by

$$T = \begin{array}{c} \text{State A} \\ \text{State B} \end{array} \begin{bmatrix} \text{State A} & \text{State B} \\ 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$$

- (a) What does the entry $T_{2,2}$ represent?
- (b) Given that State A has occurred, what is the probability that the next outcome will be State B?
- (c) If the initial state for the Markov process is 30% in state A and 70% in state B, find the distribution vector, X_1 , and interpret the results.
- (d) What does the entry in row 2 column 2 of the matrix T^3 represent?
- (e) After three interactions of the Markov process, what percent of those who start in State A will be in State B?

4. A group of physical fitness devotees works out in the gym every day. The workouts vary from strenuous to moderate to light. When their exercise routine was recorded, the following observation was made: of the people who workout strenuously on a particular day, 40% will work out strenuously the next day and 60% will workout moderately. Of the people who work out moderately 40% will work out strenuously on the next day and 65% will not work out lightly the next day. Of the people who working out lightly on a particular day, 30% will work out strenuously on the next day and 20% moderately.

(a) Give the transition matrix.

(b) Suppose on a particular Monday 70% will have a strenuous, 20% a moderate, and 10% a light workout. What percent will have a light or moderate workout on Wednesday?

(c) What percent of the people working out strenuously will still be working out strenuously 4 days later?

5. Three bookstores—the University Bookstore, Textbooks for Less, and A-plus Books—currently supply the books for a university. From a survey conducted at the beginning of the year, the University Bookstore had 40% of the market and each of the other bookstores had 30% of the market. Each semester the University Bookstore retains 80% of its customers but loses 10% to Textbooks for Less and the rest to A-plus Books. Textbooks for Less retains 70% of its customers but loses 5% to the University Bookstore and the rest to A-plus Books. A-Plus Books retains 75% of its customers but loses 20% to Textbooks for Less and the rest to the University Bookstore.

Assume that the entire summer business counts as a single semester.

- (a) Give the transition matrix.

- (b) What percent of the market share will Textbooks for Less have at the end of the year?

- (c) What will be the distribution of the market after 2 years?