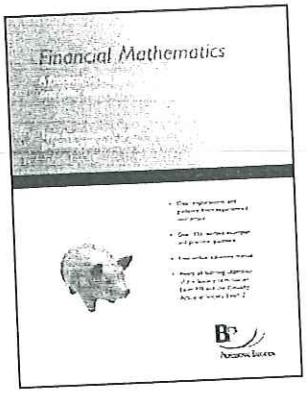

PROFESSIONAL EDUCATION

Financial Mathematics

A Practical Guide for Actuaries and other Business Professionals

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These are additional problems for our chapter 4 material.

Chapter 4 Practice Questions

Question guide

- Questions 4.1 - 4.11 test material from Sections 4.1 – 4.5.
- Questions 4.12 – 4.16 test material from Sections 4.6 – 4.7.
- Questions 4.17 – 4.20 are from the SOA/CAS Course 2 exam or the IOA/FOA 102 exam.

Question 4.1

Calculate the accumulated value, at the end of 8 years, of payments of \$4,000 a year which are paid monthly at the start of each month. The annual effective rate of interest is 9%.

Question 4.2

ch 1 material

Find the annual effective rate of interest equivalent to a nominal rate of interest of 8% a year convertible biannually.

Question 4.3

Determine the present value of payments of \$500 at the end of each year for the next 20 years. The nominal rate of interest is 5% a year convertible monthly.

Question 4.4

Find the accumulated value at time 20 years of payments of \$35 at times 0, 1, 2, and so on, with the last payment at 19 years. The nominal rate of interest is 7% a year convertible biannually.

Question 4.5

ch 1 material.

Determine the annual effective interest rate that corresponds to a nominal rate of discount of 6% a year convertible quarterly.

Question 4.6

Calculate the present value of payments of \$1,000 at the end of each year for the next 5 years. The nominal rate of discount is 10% a year convertible biannually.

Question 4.7

Find the accumulated value at time 15 years of payments of \$455 at times 0, 1, 2, and so on, until the last payment at 14 years. The nominal rate of discount is 3.5% a year convertible monthly.

Question 4.8

Determine the present value at time 0 of payments of \$4,800 a year paid at the end of each month for 7 years. The nominal rate of interest is 9% a year convertible every 4 months.

Question 4.9

Money is received at a rate of \$6,880 a year. Find the accumulated value at time 16 years of the money received, given that payments are received at the end of every other year. The nominal rate of interest is 10% a year convertible every 6 months.

Question 4.10

Calculate the present value at time 0 of payments of \$50 paid at the start of each month for 7 years, starting at time 0. The nominal rate of interest is 4% a year convertible quarterly.

Question 4.11

Find the accumulated value at time 20 years of payments of \$50,000 a year paid at the start of every half-year, starting at time 0. The nominal rate of interest is 6% a year convertible monthly.

Question 4.12

Determine the present value of payments of \$10 at the end of every month during the first year, \$20 at the end of every month during the second year, \$30 at the end of every month during the third year, and so on for 10 years. The nominal interest rate is 12% convertible monthly.

Question 4.13

Calculate the accumulated value at time 15 years of payments of \$35 at the start of every quarter during the first year, \$70 at the start of every quarter in the second year, \$105 at the start of every quarter during the third year, and so on for 15 years. The nominal interest rate is 12% convertible monthly.

Question 4.14

Find the present value of payments of \$5 now, \$10 in 6 months, \$15 in one year, \$20 in 18 months, and so on for 6 years. The nominal discount rate is 12% convertible every 6 months.

Question 4.15

Determine the accumulated value at time 5 years of payments of \$50 at the end of the first quarter, \$55 at the end of the second quarter, \$60 at the end of the third quarter, and so on for 5 years. The annual effective interest rate is 6%.

Question 4.16

Marlene invests \$X now in order to receive \$5 in 2 months, \$10 in 4 months, \$15 in 6 months, and so on. The payments continue for 10 years. The annual effective rate of interest is 8%. Determine X.

SOA/CAS

Question 4.17

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying \$10 at the end of each 3-year period, with the first payment at the end of year 6, is \$32.

At the same annual effective rate of i, the present value of a perpetuity-immediate paying \$1 at the end of each 4-month period is \$X.

Calculate X.

Question 4.18

SOA/CAS

Tawny makes a deposit into a bank account which credits interest at a nominal interest rate of 10% per annum, convertible semiannually.

At the same time, Fabio deposits \$1,000 into a different bank account, which is credited with simple interest.

At the end of 5 years, the forces of interest on the two accounts are equal, and Fabio's account has accumulated to \$Z.

Determine Z.

Question 4.19

SOA/CAS

Olga buys a 5-year increasing annuity for \$X. Olga will receive \$2 at the end of the first month, \$4 at the end of the second month, and for each month thereafter the payment increases by \$2.

The nominal interest rate is 9% convertible quarterly. Calculate X.

Question 4.20

FOA/IOA

A sum of \$100 is accumulated at a nominal rate of discount of 7.5% per year convertible quarterly for 1 year, and then at a nominal rate of interest of 7.5% per year convertible quarterly for 1 year. What is the accumulated amount of the investment after 2 years?

Chapter 4: Non-level interest rates and annuities

Q4.1: $46,236.14

Q4.2: 8.16%

Q4.3: $6,170.17

Q4.4: $1,557.76

Q4.5: 6.23%

Q4.6: $3,714.26

Q4.7: $9,138.00

Q4.8: $24,939.80

Q4.9: $240,390.22

Q4.10: $3,671.76

Q4.11: $1,959,000.90

Q4.12: $3,162.86

Q4.13: $34,972.92

Q4.14: $251.97

Q4.15: $2,191.59

Q4.16: $5,549.27

Q4.17: $39.83

Q4.18: $1,952.75

Q4.19: $2,729.21

Q4.20: $116.19

Chapter 3 Practice Questions

Question guide

- Questions 3.1 – 3.10 test material from Sections 3.1 – 3.4.
- Questions 3.11 – 3.16 test material from Sections 3.5 – 3.8.
- Questions 3.17 – 3.20 are from the SOA/CAS Course 2 exam or the IOA/FOA 102 exam.

Question 3.1

Max receives \$300 in 1 year, \$350 in 2 years, \$400 in 3 years, and so on until the final payment of \$800. Using an annual effective rate of interest of 4%, find the present value of these payments at time 0.

Question 3.2

Using the information from Question 3.1, determine the accumulated value of the payments at time 12 years.

Question 3.3

Kendra receives \$900 now, \$970 in 1 year, \$1,040 in 2 years, \$1,110 in 3 years, and so on, until the final payment of \$1,600. Using an annual effective rate of interest of 9%, find the present value of these payments at time 0.

Question 3.4

Using the information from Question 3.3, determine the accumulated value of these payments at time 11 years.

Question 3.5

Alex receives \$600 in 1 year, \$580 in 2 years, \$560 in 3 years, and so on, until the final payment of \$400. Using an annual effective rate of interest of 4%, find the present value of these payments at time 0.

Question 3.6

Using the information from Question 3.5, calculate the accumulated value of the payments at time 20 years.

Question 3.7

Rhonda receives annual payments that begin with the first payment of \$50 today. Each subsequent payment decreases by \$10 per year until time 4 years, and then each subsequent payment increases by \$10 per year until the last payment at time 8 years. The annual effective interest rate is 5%. Determine the present value of the payments at time 0.

Question 3.8

Mary is saving money for her retirement. She needs \$750,000 in 10 years to purchase a retirement apartment in Florida. She invests X now, $X-5,000$ in 1 year, $X-10,000$ in 2 years, and so on, up to $X-45,000$ in 9 years. Using an annual effective rate of interest of 5%, find X.

Question 3.9

A bank is offering a special deal. If you invest X now, the bank will give you \$15,000 at time 7 years, \$14,000 at time 8 years, \$13,000 at time 9 years, and so on, with the last payment being at time 14 years. Using an annual effective rate of interest of 8%, determine X.

Question 3.10

You invest \$500 at time 8 years, \$1,000 at time 9 years, \$1,500 at time 10 years, and so on, up to the last payment at time 20 years. What is the accumulated value of these payments at time 25 years using an annual effective rate of interest of 4.5%?

Question 3.11

Determine the accumulated value at time 10 years of payments that are received continuously over each year. The payment is \$100 during the first year, \$105 during the second year, \$110 during the third year, and so on, up to the last payment of \$145 in year 10. The annual effective interest rate is 7%.

Question 3.12

Calculate the accumulated value at time 10 years of payments that are received continuously over each year. The payment is \$200 during the first year, and each subsequent payment decreases by \$15 per year, until the last payment of \$80 is received in the ninth year. The annual effective interest rate is 4%.

Question 3.13

Brian turns 25 years old today and would like to receive inflation-adjusted retirement payments on each of his birthdays from age 65 to 95, inclusive. The first payment at age 65 will be \$100,000. The inflation rate is assumed to be 0% until age 65, and then it is assumed to be 2% per year. The annual effective interest rate is 6%. How much money should Brian set aside today to fund these future payments? Use an annuity-immediate to determine the answer.

Question 3.14

Using the same information as Question 3.13, verify the answer using an annuity-due.

Question 3.15

Calculate the present value at time 0 of a payment stream that pays $5t+1$ at time t from time 0 to 10 years. The force of interest during this time is $\delta_t = 0.01+0.05t$.

Question 3.16

A payment stream that pays $1.8t^2 + 6t$ at time t is received from time 5 to time 10 years. The force of interest from time 0 to time 5 years is $\delta_t = 0.008t + 0.03$, and the force of interest from time 5 to time 10 years is $\delta_t = 0.0003t^2 + 0.001t$. Determine the present value of the payment stream at time 0.

Question 3.17 *SOA/CAS*

Susan invests \$Z at the end of each year for seven years at an annual effective interest rate of 5%. The interest credited at the end of each year is reinvested at an annual effective rate of 6%. The accumulated value at the end of seven years is \$X.

Lori invests \$Z at the end of each year for 14 years at an annual effective interest rate of 2.5%. The interest credited at the end of each year is reinvested at an annual effective rate of 3%. The accumulated value at the end of 14 years is \$Y.

Calculate $\frac{Y}{X}$.

Question 3.18 *SOA/CAS*

Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to \$10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the current year's payment is k% larger than the previous year's payment.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of \$167.50.

Calculate k, given $k < 9.2$.

Question 3.19 *SOA/CAS*

Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$. Interest is credited at a force of interest $\delta_t = \frac{1}{8+t}$.

After 10 years, the account is worth \$20,000.

Calculate k.

Question 3.20 *FOA/IOA*

A 20-year annuity certain provides payments annually of \$200 at time 1 year, \$180 at time 2 years, \$160 at time 3 years, and so on, until the payments have reduced to \$60. Payments then continue at \$60 per year until the 20th payment has been made. The annual effective interest rate is 4%. Determine the present value of the annuity.

Chapter 3: Varying annuities

Q3.1: $4,647.00

Q3.2: $7,440.00

Q3.3: $8,831.22

Q3.4: $22,788.32

Q3.5: $4,448.74

Q3.6: $9,747.74

Q3.7: $241.14

Q3.8: $77,284.41

Q3.9: $43,099.50

Q3.10: $68,292.28

Q3.11: $1,711.49

Q3.12: $1,615.40

Q3.13: $179,451.76

Q3.14: $179,451.76

Q3.15: $92.57

Q3.16: $549.07

Q3.17: 2.0305

Q3.18: 0.04

Q3.19: 111.11

Q3.20: $1,314.39