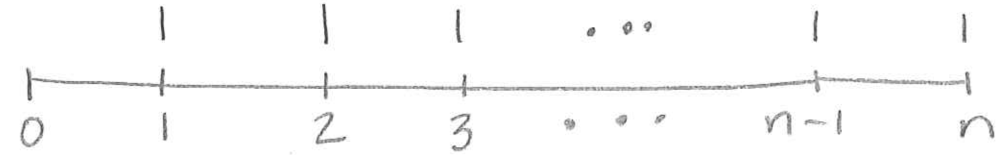
Annuities

1) $a_{\overline{n|}}$:



$$a_{\overline{n|}} = 1 \cdot v + 1 \cdot v^2 + 1 \cdot v^3 + \cdots + 1 \cdot v^n$$

$$= v + v^2 + v^3 + \cdots + v^n$$ ← (C) is correct.

$$= v(1 + v + v^2 + \cdots + v^{n-1})$$

$$= v\left(\frac{1 - v^n}{1 - v}\right)$$ ← (D) is correct.

$$= \frac{1}{1+i}\left(\frac{1 - v^n}{1 - v}\right)$$

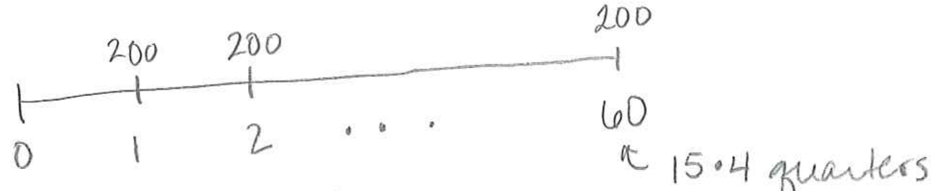
$$= \frac{1 - v^n}{1 + i - 1}$$

$$= \frac{1 - v^n}{i}$$ ← (B) is correct.

$$= v^n \left[\frac{(1+i)^n - 1}{i}\right]$$ ← (A) is correct.

$$= \frac{1}{(1+i)^n} \cdot s_{\overline{n|}}$$ ← **(E) is NOT correct.**

2)



$i^{(4)} = 4\% \Rightarrow$ Effective rate per payment period $= \frac{i^{(4)}}{4} = \frac{0.04}{4} = 0.01$

$$PV = 200\, a_{\overline{60}|0.01}$$

$$= 200\left(\frac{1 - v^{60}}{0.01}\right) \text{ where } v = \frac{1}{1.01}$$

$$= \boxed{\$8{,}991.01}$$

TVM

N = 60
I/Y = 1
PV = CPT
PMT = −200
FV = 0

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3)

$i^{(2)} = 0.06$

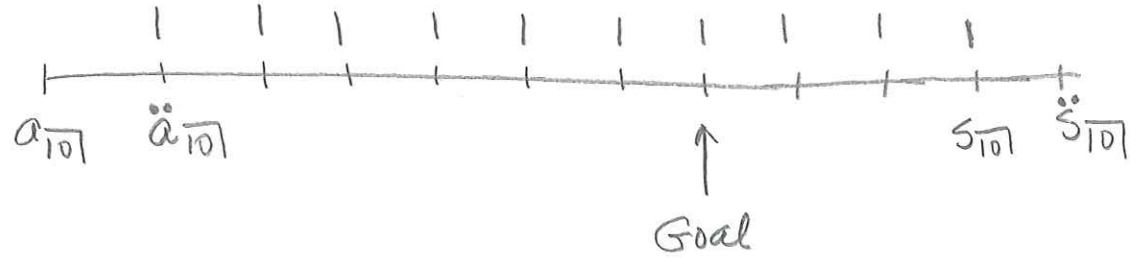
$$150\, \ddot{s}_{\overline{40}|\frac{0.06}{2}} = 150\left(\frac{(1+0.03)^{40} - 1}{\frac{0.03}{1+0.03}}\right)$$

$$= \boxed{\$11{,}649.49}$$

TVM

Set to BGN
N = 40
I/Y = 3
PV = 0
PMT = −150
FV = CPT

4)



$$a_{\overline{10}|}(1+i)^7 = \ddot{a}_{\overline{10}|}(1+i)^6 = s_{\overline{10}|}v^3 = \ddot{s}_{\overline{10}|}v^4 = s_{\overline{7}|} + a_{\overline{3}|}$$

$$= \ddot{s}_{\overline{6}|} + \ddot{a}_{\overline{4}|}$$

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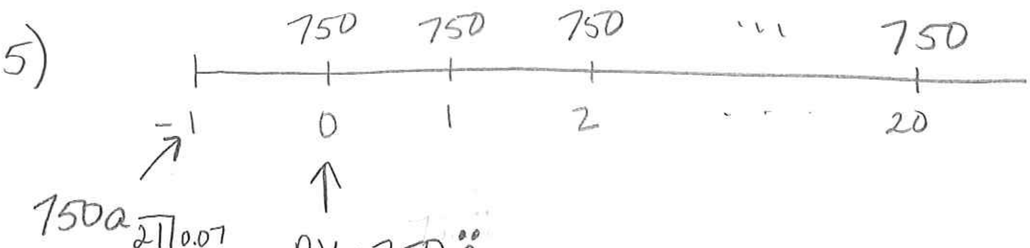
5)



$PV = 750\ddot{a}_{\overline{21}|0.07}$

As an annuity-due

Value at time 0 $= 750\ddot{a}_{\overline{21}|0.07}$

$$= 750\left(\frac{1-v^{21}}{\frac{0.07}{1.07}}\right)$$

$= \boxed{\$8695.51}$

As an annuity-immed.

Value at time 0 $= (750a_{\overline{21}|0.07})(1+0.07)$

$$= 750\left(\frac{1-v^{21}}{0.07}\right)(1.07)$$

$= \boxed{\$8695.51}$

TVM

BGN

N = 21

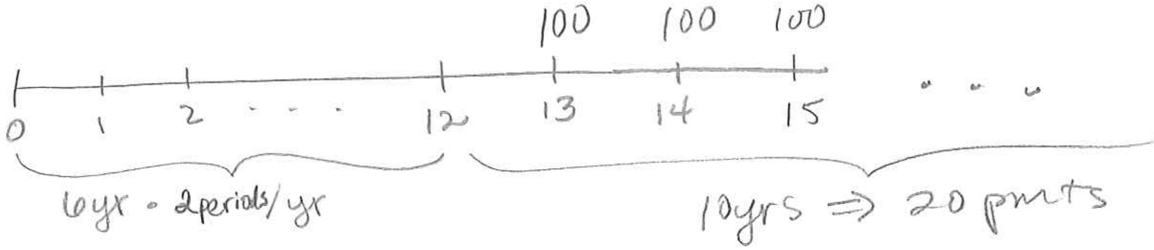
I/Y = 7

PV = CPT → \$8695.51

PMT = 750

FV = 0

6)



$$PV = 100\, a_{\overline{20}|\frac{0.10}{2}} \cdot v^{12}$$

$$= 100\left(\frac{1 - v^{20}}{0.05}\right)\left(\frac{1}{1.05}\right)^{12}$$

$$= \boxed{\$693.94}$$

TVM

Step 1 - Find PV at time 12.

N = 20

I/Y = 10 / 2 = 5

PV = [CPT] → \$1246.22

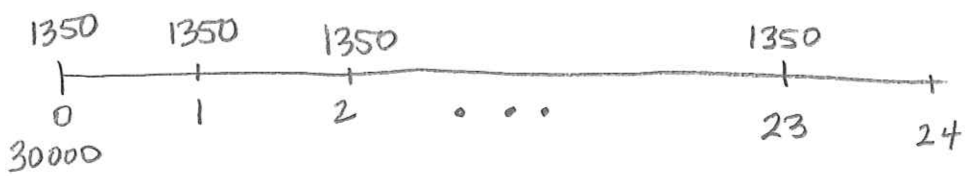
PMT = 100

FV = 0

Step 2 - Discount 12 semiannual periods:

$$1246.22\, v^{12} = \boxed{\$693.94}$$

7)



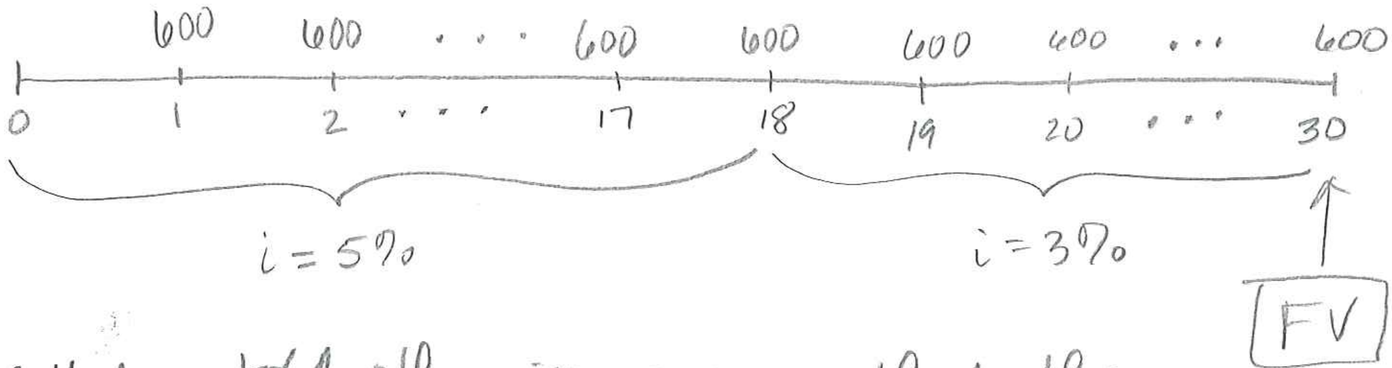
TVM ⊛ Set to [BGN] ⊛

N = 24 I/Y = CPT PV = -30000 PMT = 1350 FV = 0

$$\downarrow$$

$$0.6811336606 = \frac{i^{(12)}}{12} \Rightarrow \boxed{i^{(12)} = 8.1736\%}$$

8)



<u>Note</u>: Unless told otherwise, assume that the <u>portfolio method</u> is being used.

$$FV = 600\, s_{\overline{18}|0.05}(1+0.03)^{12} + 600\, s_{\overline{12}|0.03}$$

$$= 600\left[\frac{(1+0.05)^{18}-1}{0.05}\right](1.03)^{12} + 600\left[\frac{(1+0.03)^{12}-1}{0.03}\right]$$

$$= 24066.03223 \quad + \quad 8515.217737$$

$$= \boxed{\$32{,}581.25}$$

Step 1: Convert interest rate.

9) Payments are quarterly, so we need $i^{(4)}$.

Given $i = 0.0825$, $i^{(4)}$ can be found as follows:

$$1+i = \left(1+\frac{i^{(4)}}{4}\right)^4$$

$$(1+0.0825)^{\frac{1}{4}} - 1 = \frac{i^{(4)}}{4}$$ ← this is what we use in the formula (or in TVM, but as a percent)

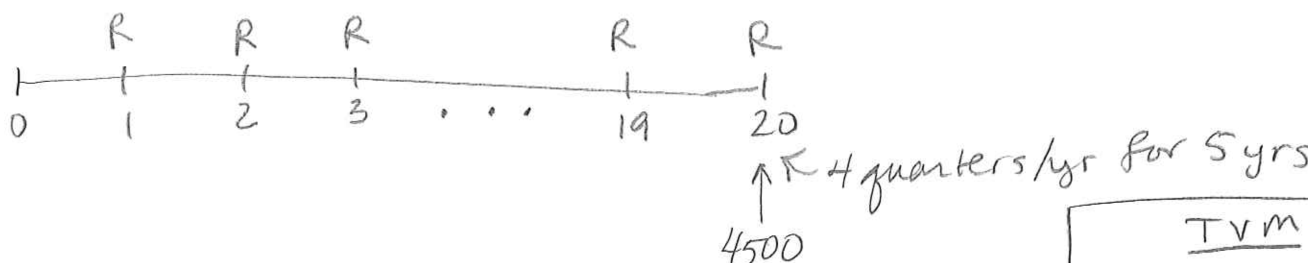
$$4\left[(1.0825)^{\frac{1}{4}} - 1\right] = i^{(4)}$$

$$i^{(4)} = 0.0800639256$$

Alternatively, use ICONV in calc:

Set EFF = 8.25 and C/Y = 4, then scroll to NOM and press [CPT]

Step 2: Time Diagram and Equation of Value



$$R s_{\overline{20}|\frac{i^{(4)}}{4}} = 4500$$

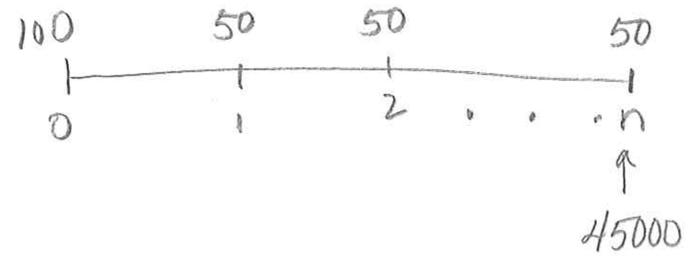
$$R = \frac{4500}{\frac{\left(1+\frac{i^{(4)}}{4}\right)^{20} - 1}{i^{(4)}/4}} = \boxed{\$185.18}$$

TVM

N = 20
I/Y = 2.001598141
PV = 0
PMT = [CPT] → [$185.18]
FV = 4500

Interest earned = 4,500 − (185.18)(20) = [$796.40]

10) $i^{(12)} = 5.25\%$



Using $t=0$ as comparison date:

$$45000v^n = 100 + 50a_{\overline{n}|\frac{0.0525}{12}}$$

$$45000v^n = 100 + 50\left(\frac{1-v^n}{\frac{0.0525}{12}}\right)$$

$$45000v^n = 100 + 11428.57143(1-v^n)$$

$$45000v^n = 100 + 11428.57143 - 11428.57143v^n$$

$$56428.57143v^n = 11528.57143$$

$$v^n = 0.2043037975$$

$$n = \ln(0.2043037975)/\ln\left(\frac{1}{1+\frac{.0525}{12}}\right)$$

$$n = 363.798568 \text{ months}$$

Note: This same value of n can be obtained very quickly with the TVM worksheet:

TVM

N = CPT → 363.798568 months

I/Y = 5.25/12

PV = −100 (initial deposit)

PMT = −50 (monthly deposits)

FV = 45,000

cont'd →

#10 continued

We found n= 363.798568 payments, which means it will take 364 payments of 50 for the account to have at least \$45,000.

364 monthly pmts = $30\frac{1}{3}$ years = 30 years and 4 months

Amt in acct after 364 pmts $= 100\left(1+\frac{0.0525}{12}\right)^{364} + 50\, s_{\overline{364}|\frac{.0525}{12}}$

$= \boxed{\$45,049.64}$

or TVM

N= 364 I/Y= 5.25/12 PV= -100 PMT= -50 FV= CPT

↓

$\boxed{\$45,049.64}$