Math 325 - Varying Annuities

- Present value of an m-thly payable annuity: $PV = (\text{annual payment}) \cdot a_{\overline{n}|}^{(m)} = (\text{annual payment}) \cdot \dfrac{1-v^n}{i^{(m)}}$, where n is the number of years and v uses the annual effective rate of interest

- Present value of a continuous annuity: $\bar{a}_{\overline{n}|} = \dfrac{1-v^n}{\delta}$

- Accumulated value of a continuous annuity: $\bar{s}_{\overline{n}|} = \dfrac{(1+i)^n - 1}{\delta}$

- Present value of a continuous varying annuity:
  - ⋆ $PV = \displaystyle\int_0^n f(t)v^t\,dt$, assuming a constant effective rate of interest i per period and payment at exact moment t given by $f(t)dt$
  - ⋆ $PV = \displaystyle\int_0^n f(t)e^{-\int_0^t \delta_r dr}dt$, assuming a varying force of interest δ_t and payment at exact moment t given by $f(t)dt$

- Present value of an annuity-immediate increasing (or decreasing) in arithmetic progression:

$$PV = Pa_{\overline{n}|} + Q\left[\frac{a_{\overline{n}|} - nv^n}{i}\right]$$

  where P is the amount of the first payment and Q is the amount by with each subsequent payment increases. (Note: Q can be negative.)

  - ⋆ If $P = 1$ and $Q = 1$, then this is referred to as an *increasing annuity* and $(Ia)_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$.
  - ⋆ If $P = n$ and $Q = -1$, then this is referred to as an *decreasing annuity* and $(Da)_{\overline{n}|} = \dfrac{n - a_{\overline{n}|}}{i}$.

- Present value of a perpetuity-immediate increasing in arithmetic progression:

$$PV = \frac{P}{i} + \frac{Q}{i^2}$$

  where P is the amount of the first payment and $Q > 0$ is the amount by which each subsequent payment increases.

- Present value of an annuity-immediate in which first payment is 1 and successive payments increase (or decrease) in geometric progression with common ratio $1+k$:

$$PV = \frac{1 - \left(\frac{1+k}{1+i}\right)^n}{i-k}$$

- Present value of a perpetuity-immediate in which first payment is 1 and successive payments increase (or decrease) in geometric progression with common ratio $1+k$, where $k < i$:

$$PV = \frac{1}{i-k}$$

NOTE: All of the above formulas assume that i is an effective rate per period. Also, formulas for accumulated values and for annuities-due can be obtained by computing present values for annuities-immediate and then accumulating that quantity to the appropriate date.

1. Find the present value of a 15-year decreasing annuity-immediate paying 150,000 the first year and decreasing by 10,000 each year thereafter. The effective annual interest rate of 4.5%. **Answer: 946,767.616**

2. An investor is considering the purchase of 500 ordinary shares in a company. This company pays dividends at the end of each year. The next payment is one year from now and it is \$3 per share. The investor believes that each subsequent payment per share will increase by \$1 each year forever. Calculate the present value of this dividend stream at a nominal rate of interest of 6.8% per annum compounded semiannually. **Answer:\$126,236.78**

3. The force of interest at time t is $\delta_t = \frac{t^3}{10}$. Find the accumulated value of a four-year continuous annuity which has a rate of payments at time t of $5t^3$. Give an exact answer. **Answer:** $50e^{6.4} - 50$

4. An annuity provides for 10 annuals payments, the first payment one year from now being \$2600. The payments increase in such a way that each payment is 3% greater than the previous one. The annual effective rate of interest is 4%. Find the present value of this annuity. **Answer: \$23,945.54**

5. Find the accumulated value of an annuity-due in which annual payments of \$43,000 paid monthly for 10 years are made to an account paying an annual effective rate of discount of 5.6%. **Answer: \$582,967.19**

6. An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are 6750, and medical inflation is expected to be 3.25% per year. The claimant is expected to live an additional 16 years. Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Find the present value of the obligation if the annual interest rate is 5.8%. **Answer: \$88,329.18**

7. An annuity pays 1 at the end of the first year. The payments increase by 2% for the next 7 years, the payments increase by 9% for the following 10 years, and then the payments increase by 11% forever. Calculate the present value of this annuity given an annual effective interest rate of 15%. **Answer: 17.95909**

8. An annuity-immediate paying X at the end of first year, with each subsequent payment decreased by 50 for the following 17 years (for a total of 18 payments), has a present value of 35,985. If $i = 0.7\%$, calculate X. **Answer: 2,550.35**

9. 3000 is deposited into Fund X, which earns an annual effective rate of 7%. At the end of each year, the interest earned plus an additional 300 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%. Determine the accumulated value of Fund Y at the end of year 10. **Answer: 6,536.71**

10. A perpetuity immediate pays out 130 semiannually for the first year. From then on the payments increase by 25 every year. Given the annual effective interest rate is 4%, find the purchase price of this perpetuity. **Answer: 38,123.80**