Part I: The Ratio Test

1.

What does the Ratio Test say?

2.

Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{5^k}$ converges or diverges.

3.

Determine whether the series $\sum_{k=1}^{\infty}(-1)^k \frac{3^k k^3}{5^k}$ converges or diverges.

4.

Determine whether the series $\sum_{k=1}^{\infty}(-1)^k \frac{5^k}{3^k k^5}$ converges or diverges.

5.

Determine whether the series $\sum_{k=1}^{\infty} \frac{(2k+1)!\cos k}{k^5 10^k}$ converges or diverges.

6.

Determine whether the series $\sum_{k=1}^{\infty} \frac{k5^{2k}}{(3k+1)!}$ converges or diverges.

7.

If $a_k = 2$ and $a_{k+1} = \frac{5k - 17 \sin k}{4k+3} a_k$ determine whether $\sum a_k$ converges or diverges.

8.

If $a_k = 2$ and $a_{k+1} = (k \sin \frac{1}{2k})a_k$ determine whether $\sum a_k$ converges or diverges.

Part II: Power Series

Problem 1. What is a power series centered at a? What is the *radius of convergence* and *interval of convergence* of a power series?

Problem 2. Write $f(x) = \frac{x}{1+x}$ as a power series. What is a? What is c_k?

Problem 3. Complete the theorem: For a given power series $\sum_{k=0}^{\infty} c_k(x-a)^k$ there are only three possibilities:

(1)

(2)

(3)

Problem 4. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{(-3)^k x^k}{\sqrt{k+1}}$.

Problem 5. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{k(x+2)^k}{3^{k+1}}$.

Problem 6. Find the ROC and IOC for $\sum_{k=0}^{\infty}(-1)^k kx^k$.

Problem 7. Find the ROC and IOC for $\sum_{k=0}^{\infty}(-1)^k a_k x^k$, where $a_0 = 1$ and $a_{k+1} = \frac{k}{k+1}a_k$.

Problem 8. Find the ROC and IOC for $\sum_{k=0}^{\infty}(-1)^k k x^k$.

Problem 9. Find the ROC and IOC for $\sum_{k=0}^{\infty}(-1)^k \frac{1}{k5^k}x^k$.

Problem 10. Find the ROC and IOC for $\sum_{k=0}^{\infty} \frac{(2x-1)^k}{5^k \sqrt{k}}$.

Problem 11. Assume that $\sum_{k=0}^{\infty} c_k 4^k$ converges. What can we say about:

(1) $\sum_{k=0}^{\infty} c_k (-2)^k$.

(2) $\sum_{k=0}^{\infty} c_k (-4)^k$.

Problem 12. Assume that $\sum_{k=0}^{\infty} c_k x^k$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of:

(1) $\sum_{k=0}^{\infty} c_k$

(2) $\sum_{k=0}^{\infty} c_k 8^k$

(3) $\sum_{k=0}^{\infty} c_k (-3)^k$