Week 12 Week in Review

courtesy: David J. Manuel

(covering 11.10, 11.11, and Exam III Review)

1 Section 11.10

1. Use the definition to find the Taylor series for the given function centered at the given a value:
   (a) $f(x) = x^3 + x^2 + x$ at $a = 1$
   (b) $f(x) = \ln(x)$ at $a = 2$
2. Use known Maclaurin series to find the Macularin series for the given function:
   (a) $F(x) = \int_0^x \sin(t^2)\,dt$
   (b) $g(x) = x^2 e^{-3x}$
3. The Taylor series of a function f is $\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{12(n+1)}$. What is $f^{(4)}(3)$ (i.e., the fourth derivative of f at $x = 3$)?

2 Section 11.11

1. Find the third degree Taylor Polynomial of $g(x) = \left(1 - \frac{x^2}{2}\right) e^x$ centered at $a = 0$.
2. Find the second degree Taylor Polynomial of $f(x) = \sqrt{x}$ centered at $a = 4$.

3 Exam III Review

1. Determine if the following series converge or diverge. If the series converges, determine whether the series converges absolutely or not. Clearly show all conditions are met for the test you use.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin(n)}{n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(3)}{n^2+1}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

2. Given the series $\displaystyle\sum_{n=1}^{\infty} ne^{-3n}$, estimate the maximum possible error when using S_5 to approximate the sum of the series.

3. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n(x-1)^n}{\sqrt{2n+3}}$

4. Given $f(x) = \dfrac{1}{1+x^2}$:

   (a) Find a power series for f .

   (b) Use your answer to (a) to find a power series for $F(x) = \arctan(x)$.

   (c) Determine the minimum number of terms N needed to guarantee that the Nth partial sum of the series in part (b) approximates the series to within $\dfrac{1}{100}$.