Final Week in Review

courtesy: David J. Manuel

(covering Final Exam Review)

1. Compute each of the following integrals:
   (a) $\displaystyle\int \frac{x-2}{x(x^2+1)}\,dx$
   (b) $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{\sqrt{x^2-1}}\,dx$
   (c) $\displaystyle\int \cos^3(2x)\,dx$
   (d) $\displaystyle\int x\sin(2x)\,dx$
   (e) $\displaystyle\int_0^{\ln(3)} \frac{e^x}{\sqrt{e^x+1}}\,dx$
2. Compute $\displaystyle\int_0^{\infty}\left(\frac{2}{2x+1}-\frac{1}{x+3}\right)dx$or show it diverges.
3. Find the area of the region bounded by the graphs of $x=0$, $y=\dfrac{3}{2}\tan x$, and $y=\cos x$.
4. The region bounded by $y=4-x^2$ and $y=3$ is revolved around the line $x=2$. Write an integral to find the volume.
5. Find the volume of the solid whose base is the triangular region with vertices (0, 0), (3, 0), and (0, 4) and whose cross-sections perpendicular to the x-axis are semicircles.
6. Consider a trough in the shape of a half-cylinder of radius 3 feet and length 8 feet (diameter at the top). It is full of water to a depth of 3 feet. Find an integral that gives the work necessary to pump all of the water to a point 1 foot above the top of the trough.
7. Write a power series for the function $f(x)=\ln(1+2x)$ centered at $x=0$.

8. Write a power series for the function $f(x) = e^{-x}$ centered at $x = 1$.

9. Determine whether the following series converge or diverge. Name and apply an appropriate test and state all the conditions that must be satisfied. If the series converges, determine if it converges absolutely.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{\sqrt{n^5 + 10}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$

10. Find the radius and interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n+7}}$.

11. Write a power series for $f(x) = e^{-x^2}$ and use it to approximate $\displaystyle\int_0^1 e^{-x^2}\,dx$ to within $\dfrac{1}{100}$.

12. Estimate the error in using s_5 to approximate the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.

13. Find the second degree Taylor polynomial for $f(x) = \sqrt{x}$ at $x = 1$.

14. The curve parametrized by $x = 3t - t^3$, $y = 3t^2$, $t \in [0, 1]$ is rotated about the x-axis. Find the area of the surface formed.

15. Sketch the graph of the polar equation $r = 8 + 8\sin(\theta)$.