Week In Review #4 – Test 1 Review
Exam 1 covers Chapter 1 (except Section 1.4) and Section 4.7.

♦ This review gives one or two examples from each section. It is NOT a thorough review by itself, but rather some additional practice problems that you can study along with your homework and lecture notes.

♦ The problems in Week-In-Review 1, 2 and 3 are also good review problems for the exam.
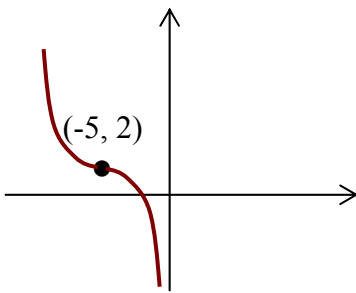
---

1. Given the graph of $f(x)$ below:
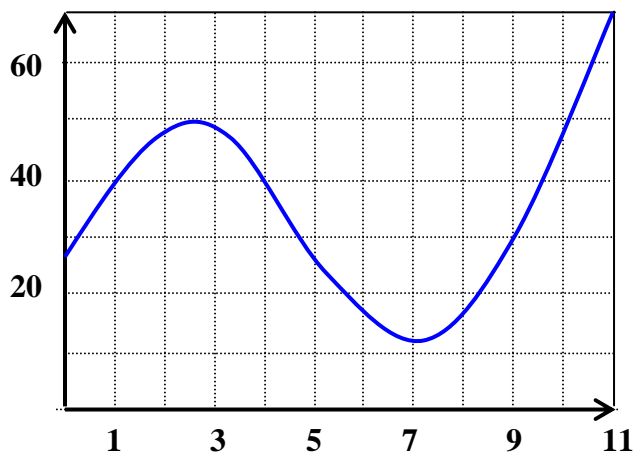
(a) Find the slope of the secant line between $x = 1$ and $x = 5$.

(b) Find the intervals on which $f(x)$ is increasing and concave down.

(c) What is the vertical intercept? What are the horizontal intercepts?

(d) During which of the following x intervals is the average rate of change the largest?

$$0 \le x \le 4 \qquad 6 \le x \le 11 \qquad 5 \le x \le 9$$

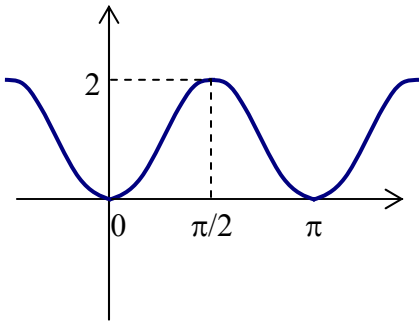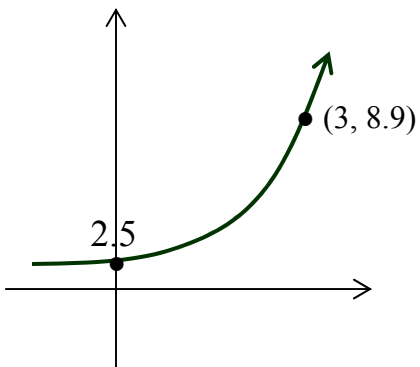Find possible formulas for the following graphs:

2.
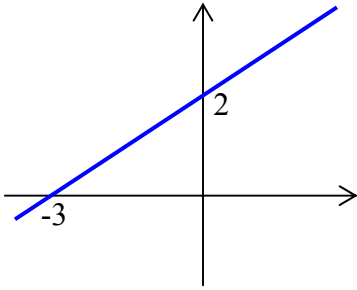
3.

4.

5.

6. Write a possible formula representing each case described below.

(a) A town has a population of 5000 people at time $t = 0$ and the population increases 150 people a year.

(b) An investment, I, is compounded continuously at an annual rate 5.6%.

(c) A drug is injected into a patient's bloodstream. The quantity, Q, in the blood increases by 0.05 mg per minute.

(d) S is proportional to the square of w and inversely proportional to five times the cube root of t.

(e) The water level, L, on a pier at Galveston is recorded hourly. When $t = 0$, there it is middle tide.

(f) A town has a population of 5000 people at time $t = 0$ and the population is increasing 15% per year.

(g) An investment, I, is compounded annually at an annual rate of 5.6%.

7. Given function $P = 7.510(.8164)^t$, what is the annual percent rate? What is the continuous percent rate?

8. The high and low tides of water differ by 5 feet at a certain pier. The middle tidemark is 12 feet from the bottom of the pier. If high tide is at midnight and low tide is 12 hours later, find a function of depth of the water with respect to time in hours since midnight.

9. Iodine-131 is a radioactive substance used as a tracer for medical diagnosis and has a half-life of 8 days. Suppose 20 units of iodine-131 are needed in two days. Determine how much should be ordered.

10. Given the graph of the polynomial below, answer the following questions:
    (a) What is the smallest possible degree for the polynomial?
    (b) Is the polynomial even or odd?
    (c) Is the leading coefficient of the polynomial positive or negative?
    (d) Is the graph below a local view or a global view for the polynomial?
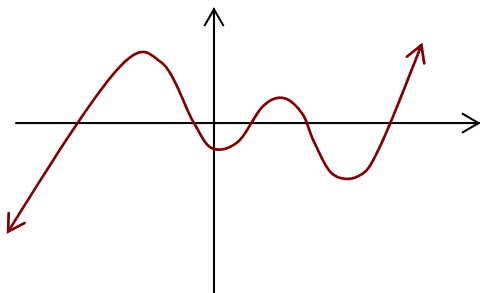
11. The total number of HIV infections increased worldwide approximately exponentially from 2.5 million in 1985 to 58.0 million in 2000.
(a) Find a formula for the number of HIV infections, H, (in millions) as a function of the number of years, t, since 1985.

(b) What was the continuous annual percent rate in the number of HIV infections between 1985 and 2000?

(c) What was the annual percent rate in the number of HIV infections between 1985 and 2000?

(d) What was the average rate of change in the number of HIV infections between 1985 and 2000?

12. What interest rate would allow an account to double over 10 years, if the interest is compounded continuously?

13. After 14 days, a chemical, which decays exponentially, has lost 18% of its original amount. What is the half-life of this chemical?

14. The body weight, W (in lbs.), of a whale is inversely proportional to the cube of the age, A, (in years) of the whale and directly proportional to the square root of the length, L, (in feet) of the whale. If the whale is 2 years old, weighs 480 lbs., and is 22 feet long, what is the constant of proportionality?

15. If $h(t) = -16t^2 + 100t + 6$ is the height of a ball thrown upwards at 100ft/s, then answer the following questions:
(a) What is the average velocity from $t = 2$ to $t = 4$?
(b) What are the initial height and the maximum height for the ball?

16. Sketch a possible graph of a function that is decreasing everywhere, concave down for $x < 3$ and concave up for $x > 3$.

17. Sketch the graph of a polynomial function that satisfies all of the following:

(a) It is an even function.
(b) It has zeros at $x = -1$, $x = 0$, and $x = 3$.
(c) The leading coefficient is negative.

Is the graph you sketched above a local view (in a small window) or a global view (in a large window) for the polynomial?

18. Simplify the expression as much as possible: $\log_a \dfrac{(2x+3)^5}{a^2 \cdot \sqrt[3]{x-2}}$

19. Solve for x:

(a) $3 \cdot e^{1.8x} = 7 \cdot e^{2x}$

(b) $\log_3 x + \log_3 (x-2) = 1$

20. If $m(x) = 1 - x^2$, and $g(x) = \dfrac{1}{x}$, find:

(a) $m(g(2))$

(b) $m(x+h) - m(x)$

21. The table below gives the population of the Northeast in millions for some selected years.

Year	1790	1830	1870	1910	1950	1990
Sales	2	5.5	12.3	25.9	38.5	50.8

(a) Find the best-fitting logistic curve.
(b) Use your best-fitting logistic curve to estimate the limiting population.
(c) In what year did the rate at which the population is growing start to decrease?

Answers:

1. (a) The slope of the secant line is about -3.75.
   (b) f(x) is increasing and concave down on (0, 2.5).
   (c) The vertical intercept is about 27, no horizontal intercepts.
   (d) $6 \leq x \leq 11$

2. $y = \dfrac{2}{3}x + 2$

3. $y = -(x+5)^3 + 2$

4. $y = 1 - \cos(2x)$

5. $y = 2.5(1.527)^x$

6. (a) $P(t) = 5000 + 150t$ (b) $P(t) = Ie^{0.056t}$

   (c) $Q(t) = 0.05t$ (d) $S = k\dfrac{w^2}{5 \cdot \sqrt[3]{t}}$

   (e) $L = A\sin(Bt) + C$ (f) $P(t) = 5000(1.15)^t$

   (g) $F(t) = I(1.056)^T$

7. The annual percent rate is $r = -18.4\%$. The continuous percent rate is $k = -20.3\%$.

8. $y = 2.5\cos(\dfrac{\pi}{12}t) + 12$

9. 23.8 units

10. (a) 5   (b) odd   (c) positive   (d) local view

11. (a) $P(t) = 2.5e^{0.2096t}$ or $P(t) = 2.5(1.233)^t$
    (b) The annual continuous percent growth rate was 20.96%.
    (c) The annual percent growth rate was 23.3%.
    (d) The average rate of change of the number of HIV infections was 3.7 million people per year.

12. 6.9%
13. 48.8 days.
14. 818.7
15. (a) 4 ft/sec
    (b) The initial height is 6 ft. The maximum height is 162.25 ft.
16. Graph.
17. Graph.

18. $5\log_a(2x+3) - 2 - \dfrac{1}{3}\log_a(x-2)$

19. (a) -4.236   (b) 3

20. (a) 3/4   (b) $-2xh - h^2$

21. (a) $P(t) = \dfrac{61.1737}{1 + 26.1823e^{-0.024\,t}}$
    (b) The limiting value is about 61.17 million.
    (c) Around 1926.

If you find any mistakes, please let me know. Thanks!   *li-chen2@neo.tamu.edu*