Li Chen @Spring 2006

Week In Review #7 - Test 2 Review

Covers sections: 2.1 - 2.4, 3.1 - 3.5, 4.1 - 4.3

- This review gives one or two examples from each section. It is NOT a thorough review by itself, but rather some additional practice problems that you can study along with your homework and lecture notes.

- The problems in Week-In-Review 5 and 6 are also good review problems for the exam.

Review Problems:

1. Consider the function $f(x) = x \cdot 2^x$. (Round the answers to three decimal places.)

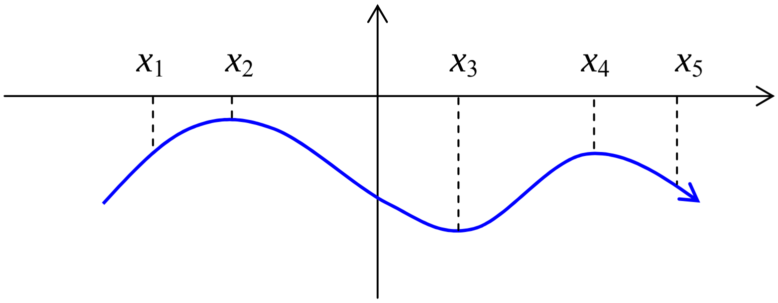
(a) Find the average rate of change of *f* between $x = 2$ and $x = 4$.

(b) Estimate $f'(3)$ by using **a** small interval.

(c) Find $f'(3)$ by using derivative formulas.

(d) Find the equation of the line tangent to $f(x)$ at $x = 3$.

2. The graph of f' is given below, at which value of x is

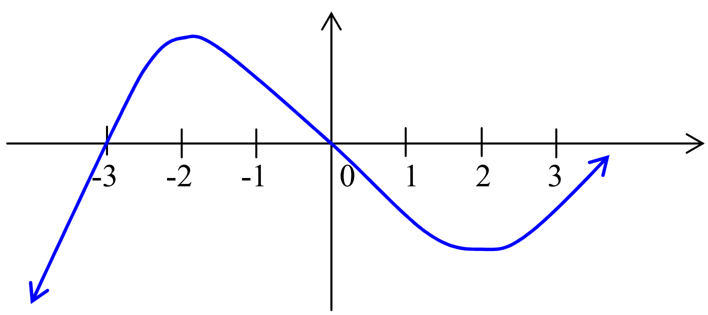


(a) $f(x)$ greatest? (b) $f(x)$ least? (c) $f'(x)$ greatest?

(d) $f'(x)$ least? (e) $f''(x)$ greatest? (f) $f''(x)$ least?

3. (a) If the graph below is of $f(x)$, which of the following has the largest value?

$f''(-2)$ $f'(3)$ $f(3)$ $f'(-3)$ $f'(0)$



(b) If the above graph is of $f(x)$, where is $f(x)$ increasing?

(c) If the above graph is of $f(x)$, where is $f(x)$ concave up?

(d) If the above graph is of $f'(x)$, where is $f(x)$ increasing?

(e) If the above graph is of $f'(x)$, where does $f(x)$ have inflection points?

(f) If the above graph is of $f'(x)$, where does $f(x)$ have local maxima?

(g) If the above graph is of $f''(x)$, where is $f(x)$ concave up?

4. Let $f(x) = 3x^3 - 5x^2 + x + 9$.

(a) Find all critical points of $f(x)$ and state what type of critical points they are (local max, local min, or either).

(b) Find the inflection point(s).

(c) Find the interval(s) at which $f(x)$ is decreasing and concave up at same time.

(d) Find the globe maximum and globe minimum of $f(x)$ on the interval [-2, 1].

5. The value of A Toyota minivan purchased in 2005 can be approximated by the function $V(t) = 25(0.85)^t$, where t is the time, in years, from the date of purchase, and V is the value, in thousands of dollars.

(a) Evaluate and interpret $V(4)$.

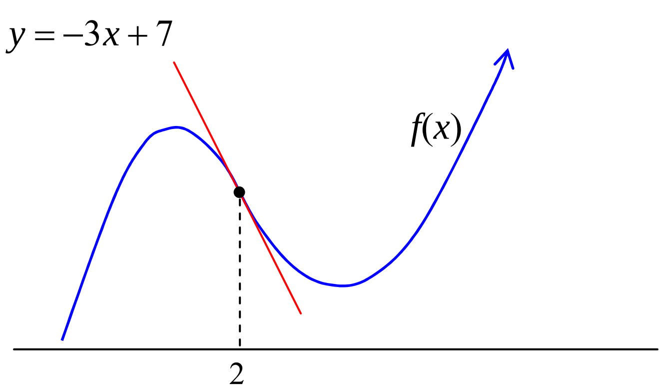
(b) Evaluate and interpret $V'(4)$.

6. The table below shows the world gold production, $G = f(t)$, as a function of year, t.

t (year)	1987	1990	1993	1996	1999
G (million troy ounces)	53	70	73	74	81

(a) Does $f'(t)$ appear to be positive or negative? What does this mean in terms of gold production?
(b) In which time interval does $f'(t)$ appear to be greatest?
(c) Estimate $f'(1999)$. Give units and interpret the answer in terms of gold production.
(d) Use the estimate value of $f'(1999)$ to estimate $f(2000)$ and (2005), and interpret the answers.

7. The following figure shows the tangent line approximation to $f(x)$ near $x = a$.
(a) Find a, $f(a)$, and $f'(a)$.
(b) Estimate $f(2.1)$ and $f(1.98)$. Are these under or overestimates? Which estimate would you expect to be more accurate and why?



8. Sketch a possible graph of a function that satisfies all of the given conditions.

$f(6)=0,\ f(0)=1,\ f'(4)=0$

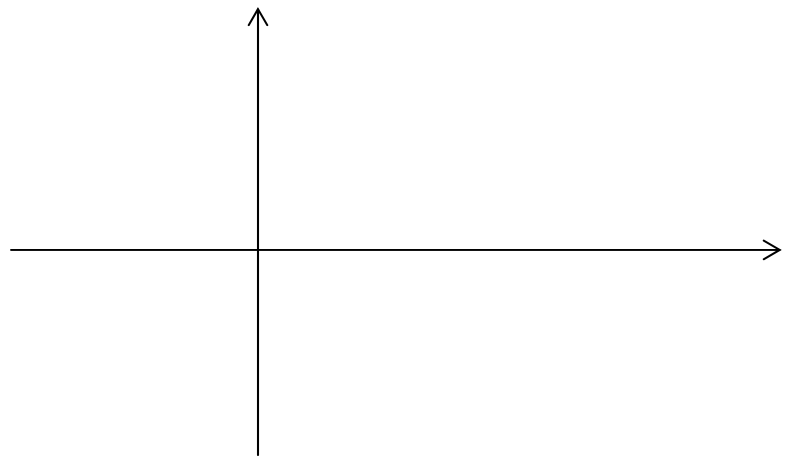
$f'(x)>0$ for $0<x<4$,

$f'(x)<0$ for $x<0$ and $x>4$,

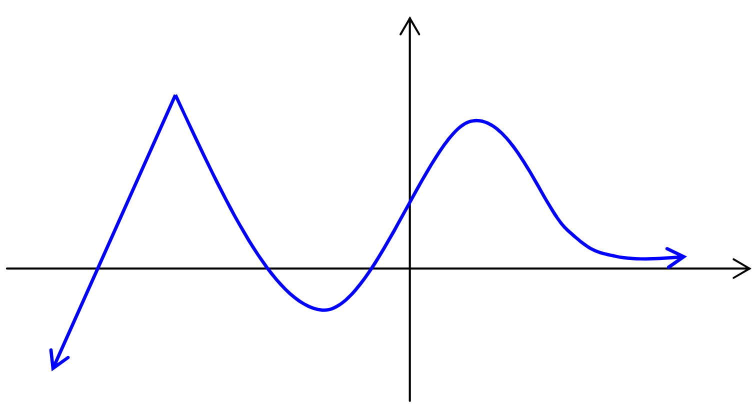
$f''(x)>0$ for $-2<x<2$,

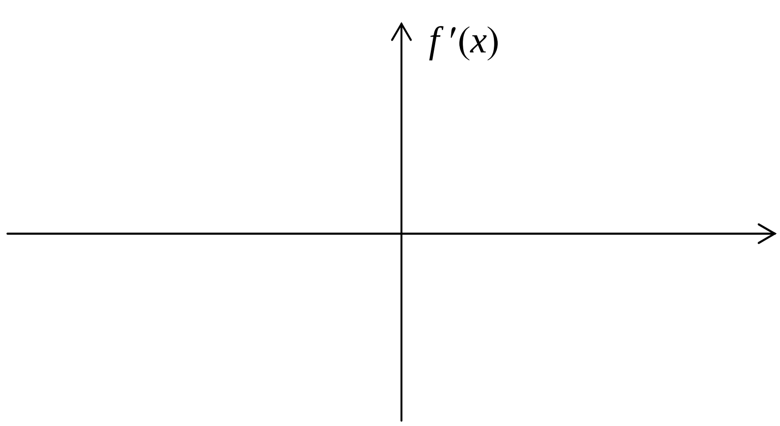
$f''(x)<0$ for $x<-2$ and $x>2$,

$f(x)\to 3$ as $x\to -\infty$,



9. The following graph is of $f(x)$. Sketch the graph of $f'(x)$.

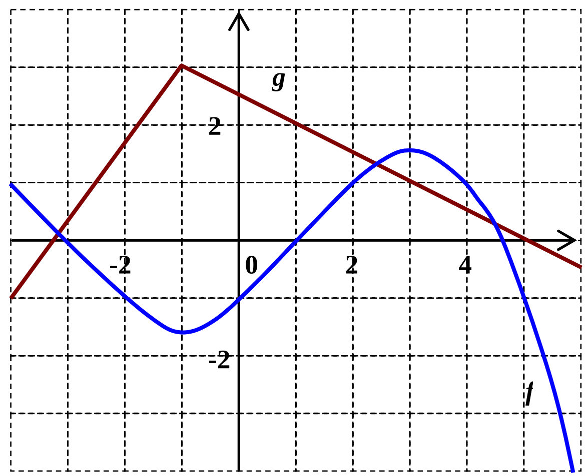




10. Find the value of a so that the function $f(x) = xe^{ax}$ has a critical point at $x = 3$.

11. If f and g are functions whose graphs shown on the right, find

(a) $h'(1)$ if $h(x) = g(f(x))$.



(b) $h(-2)$ if $h(x) = f(x^2)$.

(c) $h'(1)$ if $h(x) = 3 - f(x)g(x)$

(d) $h'(3)$ if $h(x) = \dfrac{g(x) - 2}{f(x)}$

12. During a flu outbreak in a school of 763 children, the number of infected children, *I*, was expressed in terms of the number of susceptible (but still healthy) children, *S*, by the expression $I = 192\ln\left(\frac{S}{762}\right) - S + 763$. What is the maximum possible number of infected children?

Find the derivatives for the following functions:

13. $f(x) = \frac{1}{2x} + 3\cdot\sqrt[3]{x^5} + e^{-x}$

14. $g(x) = \ln(2x) - 3\cos x + \ln 3$

15. $y = \frac{5}{2\cdot\sqrt{\ln x + xe^x + 1}}$

16. $g(t) = \dfrac{t^2 + 5t + 2}{t + 3}$

17. $H(t) = \ln \dfrac{t^2 + 5t + 2}{t + 3}$

18. $g(x) = (2 - 3x^2)^2 \sin(3x^2)$

19. Find $f''(x)$ for $f(x) = e^{x^2 + 4x}$.

Answers:

1. (a) 28 (b) 24.6367 (c) 24.6355 (d) $y = 24.6355x - 49.9$
2. (a) x_1 (b) x_5 (c) x_2 (d) x_3 (e) x_1 (f) x_5

3. (a) $f'(-3)$ is the largest value.
   (b) $f(x)$ is increasing on $(-\infty, -2)$ and $(2, \infty)$.
   (c) $f(x)$ is concave up on $(0, \infty)$.
   (d) $f'(-3)$ is the largest value.
   (e) $f(x)$ is increasing on $(-3, 0)$.
   (f) $f(x)$ has inflection points at $x = \pm 2$.
   (g) $f(x)$ has a local maximum at $x = 0$.
   (h) $f(x)$ is concave up on $(-3, 0)$.

4. (a) $f(x)$ has two critical points: $x = 1/9$ and $x = 1$.
   $f(1/9)$ is a local maximum. $f(1)$ is a local minimum.
   (b) The inflection point is at $x = 5/9$.
   (c) $f(x)$ is decreasing and concave up on $(5/9, 1)$.
   (d) The globe maximum is 9.0535 and the globe minimum is -37.

5.(a) $V(4) \approx 13.05$ which means the car will be worth about $13,050 in 2009.
   (b) $V(4) \approx -2.121$ which means the value of the car will be decreasing at a rate of $2,121 per year in 2009.
   Or in year 2010, the car will be worth about $11,040.

6. (a) $f'(t)$ appears to be positive, because according to the table, gold production is increasing.
   (b) The derivative (the rate of change) appears to be greatest between 1987 and 1990.
   (c) $f'(1999) \approx 2.333$ million troy ounces/year. In 1999, gold production was increasing at a rate of approximately 2.333 million troy ounces per year.
   (d) $f(2000) \approx 83.333$ million troy ounces $f(2005) \approx 94.998$ million troy ounces

7, (a) 2, 1, and -3. (b) $f(2.1) \approx 0.7$ (under); $f(1.98) \approx 1.06$ (over) and is better.

8, 9, Graph. 10. $a = -1/3$

11, (a) -1/2 (b) 4 (c) -2 (d) -1/3

12. 306 children

13. $f'(x) = -\dfrac{1}{2x^2} + 5x^{2/3} - e^{-x}$

14. $g'(x) = \dfrac{1}{x} + 3\sin x$

15. $y = -\dfrac{5}{4}(\ln x + xe^x + 1)^{-3/2}(\dfrac{1}{x} + e^x + xe^x)$

16. $H'(t) = \dfrac{(t+3)(2t+5) - (t^2+5t+2)}{(t+3)^2} = \dfrac{t^2+6t+13}{(t+3)^2}$

17. $H'(t) = \dfrac{2t+5}{t^2+5t+2} - \dfrac{1}{t+3}$

18. $y' = -12x(2-3x^2)\sin(3x^2) + 6x(2-3x^2)^2\cos(3x^2)$

19. $f''(x) = 2e^{x^2+4x}\left(2x^2+8x+9\right)$

If you find any mistakes, please let me know. Thanks! *li-chen2@neo.tamu.edu*