Week In Review #10 - Test 3 Review
Covers: Chapter 5 and Chapter 7

- This review gives one or two examples from each section. It is NOT a thorough review by itself, but rather some additional practice problems that you can study along with your homework and the lecture notes.

- The problems in Week-In-Review 8, and 9 are also good review problems for the exam.

Review Problems:

1. The marginal cost $C'(q)$ (in dollars per unit) of producing q units is given in the following table.

q	0	100	200	300	400	500	600
$C'(q)$	25	20	18	22	28	35	45

(a) Interpret $C'(400)$.

(b) What are the units and meaning of $\int_0^{400} C'(q)dq$?

(c) Estimate $\int_0^{400} C'(q)dq$.

(d) If the fixed cost is $10,000, estimate the total cost of producing 400 units.

2. (a) Estimate $\int_1^4 \sqrt{x}dx$ using a left-hand-sum with $\Delta x = 0.5$.

(b) Represent the answer in (a) graphically.

(c) Use a calculator to find the value of $\int_1^4 \sqrt{x}dx$.

(d) Use the Fundamental Theorem of Calculus to find the value of $\int_1^4 \sqrt{x}dx$.

3. A baseball thrown directly upward at 96 ft/sec has velocity $v(t) = 96 - 32t$ ft/sec at time t seconds.

(a) Graph the velocity from $t = 0$ to $t = 6$.

(b) When does the baseball reach the peak of its flight? How high does it go?

(c) How high is the baseball at time $t = 5$?

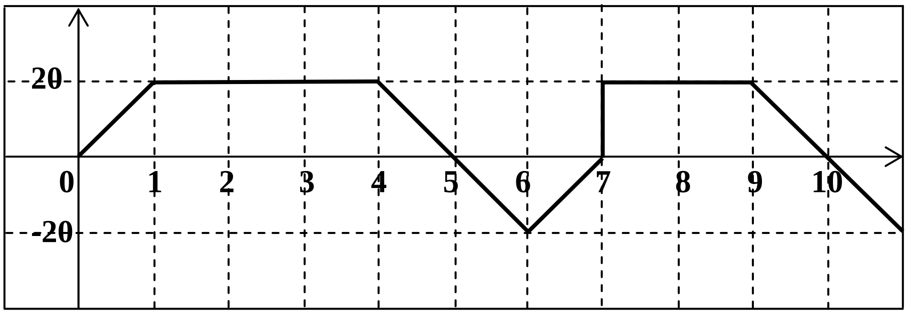
4. A bat starts out traveling towards the exit of a tunnel 90 feet away. The graph below describes the bats velocity vs. time.

(a) When does the bat change the direction?

(b) How far from the starting point is the bat after 6 seconds?

(c) How far has the bat traveled in the 6 seconds?

(d) Does the bat make it out of the tunnel in 10 seconds?

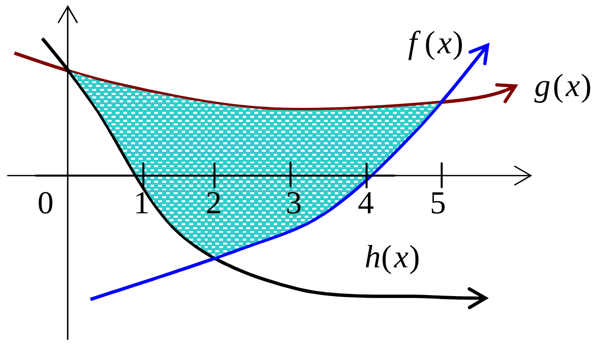


5. For the function $g(x) = \frac{3^x}{4} - 3$.

(a) Evaluate the definite integral for the given function from x = -1 to x = 5.

(b) Find the area between the function and the x-axis from x = -1 to x = 5.

6. Set up the integral(s) that you would use to find the shaded area for the following graph.



7. Determine the value of b if the area under the graph of $f(x)=\frac{2}{x}$ between $x = 1$ to x = b is 4. Assume $b > 1$.

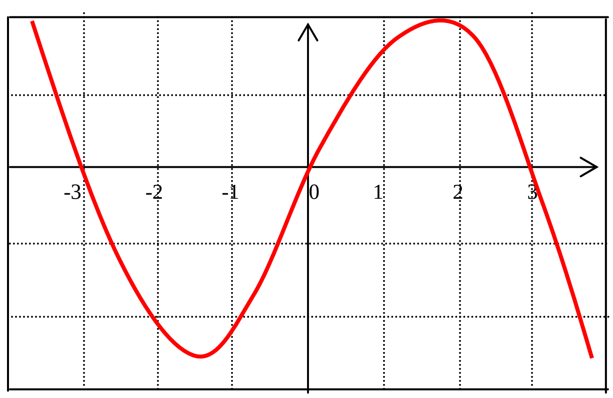
8. Consider the following graph of $f(x)$, arrange the following integrals in increasing order.

$$A=\int_{-1}^{2} f(x)dx$$

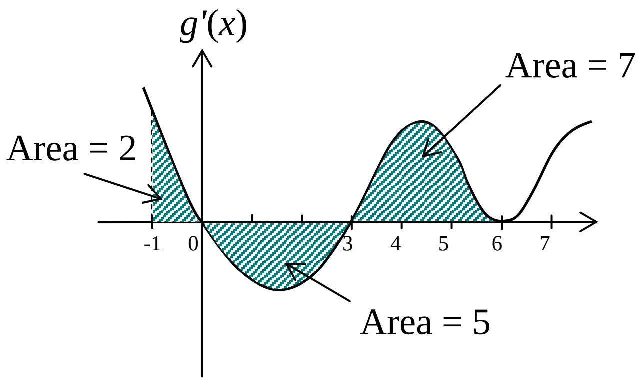
$$B=\int_{-1}^{1} f(x)dx$$

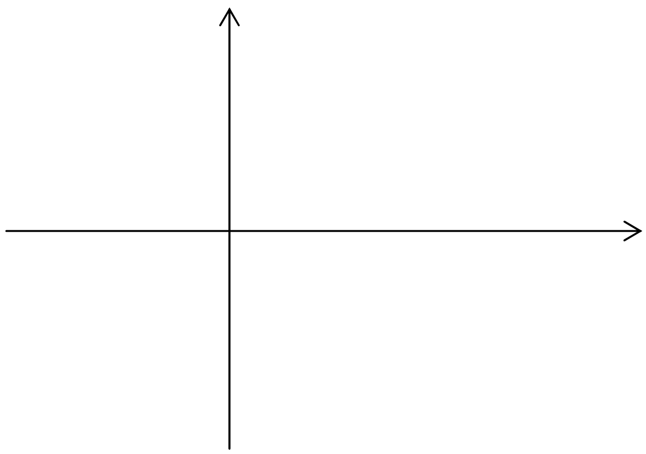
$$C=\int_{0}^{3} f(x)dx$$

$$D=\int_{-3}^{0} f(x)dx$$



9. The following graph is the graph of $g'(x)$, with some areas labeled. If $g(-1) = -3$,
(a) find (x, y) coordinates for all local maxima and minima of $g(x)$.
(b) estimate (x, y) coordinates for all inflection points of $g(x)$.
(b) Sketch the graph of $g(x)$. Be sure to label.





10. If $F'(x) = 5\sin(3x^2)$ and $F(-1) = -1$, find $F(1)$.

11. Water is pumped out of a holding tank at a rate of $f(t) = 5 - 6e^{-0.12t}$ liters/minute, where t is in minutes since the pump is started.

(a) Use a graphing calculator to find the total quantity of water pumped out during the first hour.

(b) If the tank contains 1000 liters of water when the pump is started, how much water does it hold one hour later?

(c) Use the Fundamental Theorem of Calculus to find the total quantity of water pumped out during the first hour.

Find the following indefinite integrals:

12. $\int (x^2 + \frac{1}{3x^3} - e)dx$

13. $\int (4 + e^{-4x} - \cos(4x))dx$

14. $\int \frac{\sqrt{y} + 2y}{y^2} dy$

15. $\int (x^3+2)\sqrt[3]{x^4+8x+3}\,dx$

16. $\int \frac{1}{x\ln x}dx$

17. $\int \frac{e^{1/x}}{x^2}\,dx$

18. $\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^3}\, dx$

19. $\int \sin^6(5\theta)\cos(5\theta)\, d\theta$

20. Evaluate the definite integral EXACTLY: $\int_0^1 t^2 e^{-t^3}\, dt$

Answers:

1. (a) $C'(400) = 28$ means that when 400 units are produced, the cost is increasing at a rate of 28 dollars per unit.

(b) The units of $\int_0^{400} C'(q)dq$ are dollars. $\int_0^{400} C'(q)dq$ equals the variable cost when 400 units are produced.

(c) \$8,650. (d) \$18,650.

2. (a) LHS = 4.4115 (b) 4.6667

3. (b) 3 sec, 144feet (c) 80 ft

4. (a) $t = 5$, 7, 10 seconds (b) 70 ft (c) 90 ft (d) Yes.

5. (a) 37.22 (b) 51.48

6. The area $= \int_0^2 [g(x) - h(x)]dx + \int_2^5 [g(x) - f(x)]dx$

7. $b = e^2$

8. D < B < A < C

9. (a) Local maximum: (0, -1)
Local minimum: (3, -6)
$g(x)$ has neither at $x = 6$.

(b) Inflection points: (1.5, -3.5), (4.5, -2.5), (6, 1)

10. 4.15

11. (a) 250 litters (b) 750 litters

12. $\frac{1}{3}x^3 - \frac{1}{6}x^{-2} - ex + C$

13. $4x - \frac{1}{4}e^{-4x} - \frac{1}{4}\sin(4x) + C$

14. $-\frac{2}{\sqrt{y}} + 2\ln|y| + C$

15. $\frac{3}{16}(x^4 + 8x + 3)^{4/3} + C$

16. $\ln|\ln x| + C$

17. $-e^{1/x} + C$

18. $-\frac{1}{2(e^x + e^{-x})^2} + C$

19. $\frac{1}{35}(\sin(5\theta))^7 + C$

20. $\frac{1}{3}(1 - \frac{1}{e})$

If you find any mistakes, please let me know. Thanks! ***li-chen2@neo.tamu.edu***