

Exam # 2 Sample Review
 Sections 2.1-2.4, 3.1-3.5, and 4.1-4.3

1. Let $f(x) = 5x^3 - 8x^2 + x + 70$

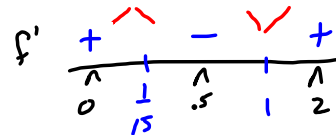
(a) Find all critical values of $f(x)$ and classify them as local max, local min, or neither.

$f'(x) = 15x^2 - 16x + 1 = (15x-1)(x-1)$ *domain is all Real #s.*

$0 = (15x - 1)(x - 1)$

C.V. $x = \frac{1}{15}$ $x = 1$

Local max at $x = \frac{1}{15}$



Local min at $x = 1$

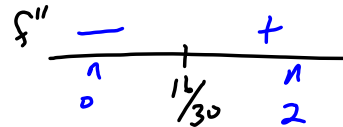
(b) Find the inflection values.

$f'' = 30x - 16$

$0 = 30x - 16$

$16 = 30x$

$x = \frac{16}{30}$



Inflection point at $x = \frac{16}{30}$

(c) Find the interval(s) where $f(x)$ is decreasing and concave down.

$(\frac{1}{15}, \frac{16}{30})$

(d) Find the interval(s) where $f(x)$ is increasing and concave up.

$(1, \infty)$

(e) Find the global maximum and global minimum of $f(x)$ on the interval $[0, 6]$

C.V. $x = \frac{1}{15}$ $x = 1$

$f(\frac{1}{15}) = 70.033$

$f(1) = 68$

$f(0) = 70$

$f(6) = 868$

*$f(x)$ is cont.
on the closed
interval $[0, 6]$*

global max = 868

global min = 68

2. Let $f(x) = 2x^4 - 10x^3 + 10$.

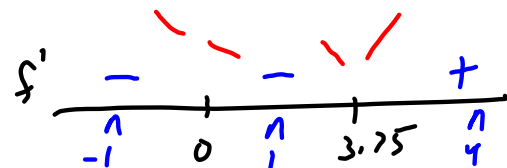
(a) Find the intervals where $f(x)$ is increasing and where it is decreasing. Classify the critical values.

$$f'(x) = 8x^3 - 30x^2$$

$$= x^2(8x - 30)$$

$$0 = x^2 \quad 0 = 8x - 30$$

$$x = 0 \quad x = \frac{30}{8} = 3.75$$



Inc $(3.75, \infty)$
 dec $(-\infty, 0)$ $(0, 3.75)$

Local min at $x = 3.75$
 $x = 0$ is a neither.

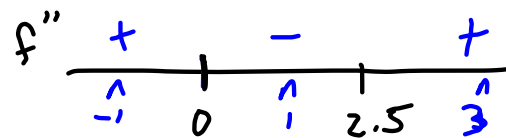
(b) Find the intervals where $f(x)$ is concave up and where it is concave down. Find the x-values of the inflection points.

$$f''(x) = 24x^2 - 60x$$

$$= 12x(2x - 5)$$

$$0 = 12x \quad 0 = 2x - 5$$

$$x = 0 \quad x = \frac{5}{2} = 2.5$$



c.u. $(-\infty, 0)$
 $(2.5, \infty)$

Inflection point
 at $x = 0$ + $x = 2.5$

c.d $(0, 2.5)$

2. Let $f(x) = 2x^4 - 10x^3 + 10$.

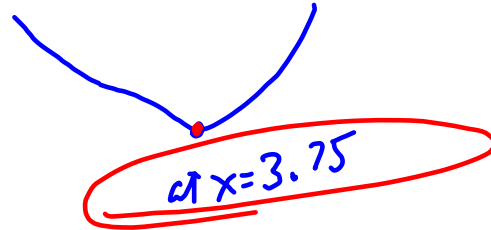
even poly. that opens up so no global max.

(c) Does the function have a global max? If so, where?

(d) Does the function have a global min? If so, where?

by this: yes

Look at part A for shape.



(e) Does the function have a global max on the $[1, 4]$? If so, where?

cont. function closed interval so yes.

$$f(1) = 2$$

$$f(4) = -118$$

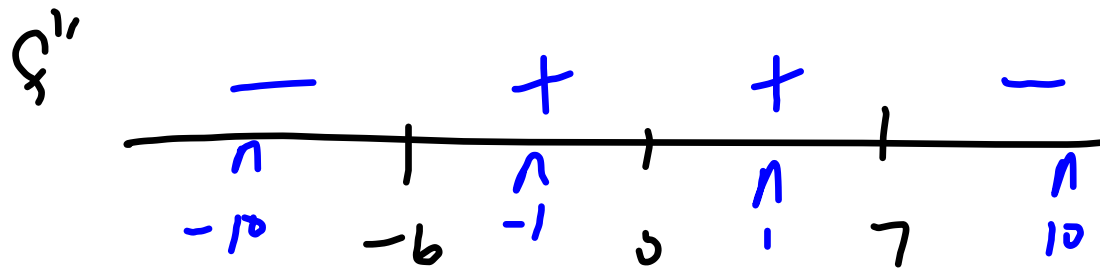
$f(3.75)$ ← by part d global min.

C.V. $x=0$
 $x=3.75$
Not in the interval.

global max at $x=1$

3. Find the x-values of the inflection points for $f(x)$ if the domain of $f(x)$ is all real numbers and $f'''(x) = 3x^2(x + 6)(7 - x)^3$

$$0 = f'''(x) \quad (7-x)^3 = 0 \quad 7-x=0$$
$$x=0 \quad x=-6 \quad x=7 \quad 7=x$$



Inflection point at $x = -6$ + $x = 7$

4. Find the constants a and b such that the point $(1, -4)$ is an inflection point for $f(x) = ax^3 + bx^2 + 10$.

$$f'(x) = 3ax^2 + 2bx$$

$$f''(x) = 6ax + 2b$$

$$f(1) = -4$$

$$a + b + 10 = -4$$

$$a + b = -14$$

$$a - 3a = -14$$

$$-2a = -14$$

$$a = 7$$

$$f''(1) = 0$$

$$6a + 2b = 0$$

$$2b = -6a$$

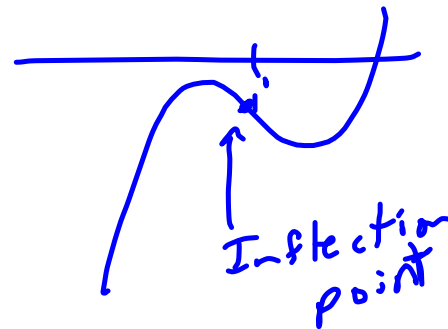
$$b = -3a$$

$$b = -21$$

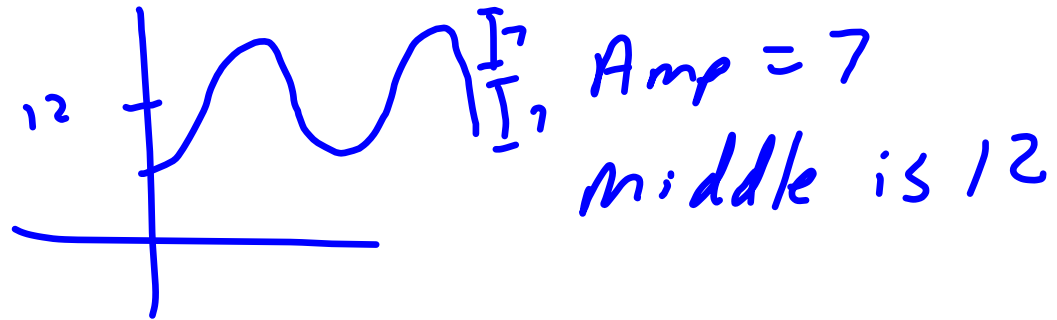
$$y = 7x^3 - 21x^2 + 10$$

5. For problem 4, is the critical value that is less than 1 a local max or a local min? Answer this question without performing any calculations or using the graphing calculator.

Local
max



6. Find the global max and the global minimum for $y = 12 - 7 \cos(3x)$



$$\text{global max} = 12 + 7 = 19$$

$$\text{global min} = 12 - 7 = 5$$

7. Give the formula for a function that has a global maximum but doesn't have a global minimum.

$$y = -x^2$$

$$\text{or } y = -32x^{40} + 3x + 1$$

8. Let $f''(x) = 6x - 15$. What can be concluded about the following values of x .

(a) $x = 3$

$$f''(3) = 18 - 15 = 3$$

Concave up.

(b) $x = 5$ if $f'(5) = 0$

$$f''(5) = 30 - 15 > 0$$

$x = 5$ is a c.v.

concave up.

Local min at $x = 5$

(c) $x = 0$ if $f'(0) = 0$

$$f''(0) = -15$$

c.v.

concave down

Local max at $x = 0$

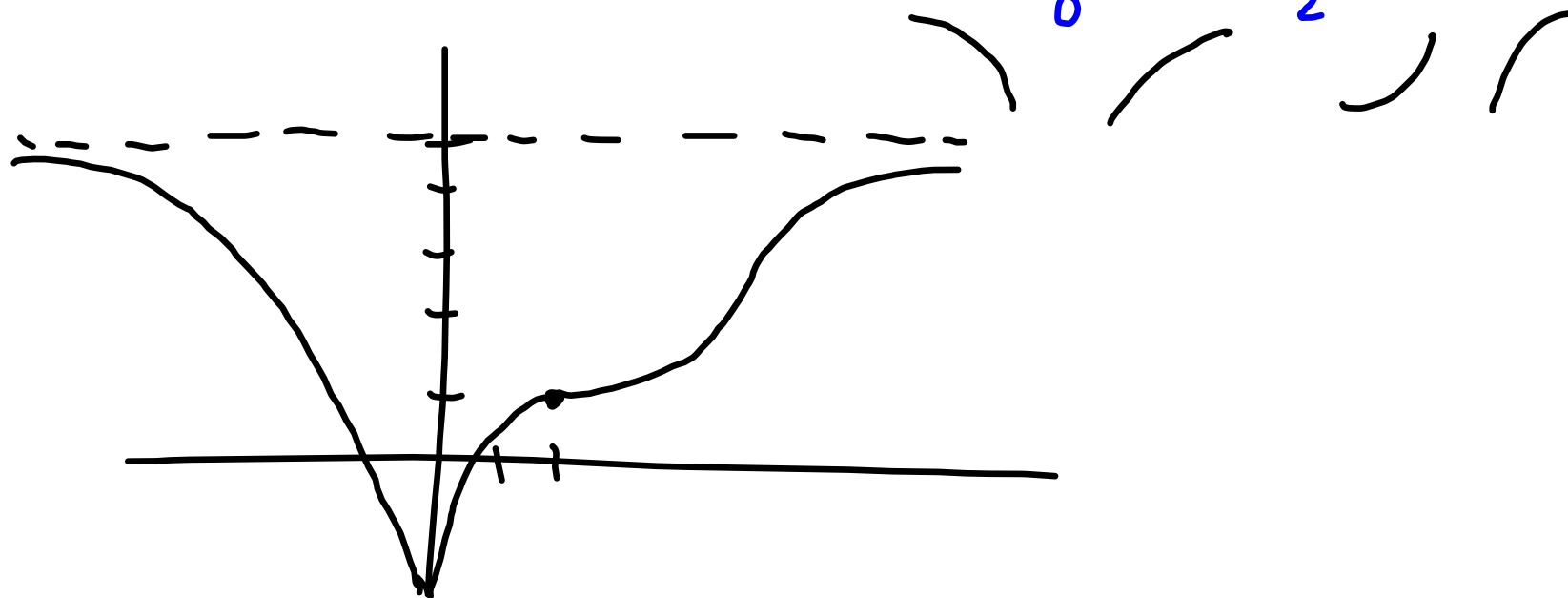
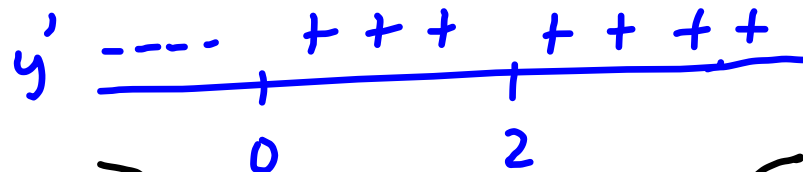
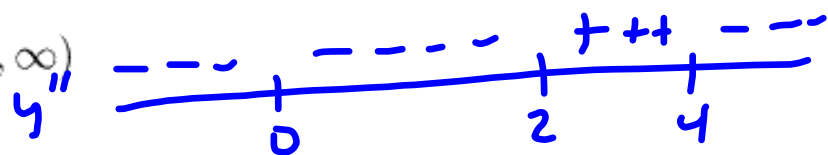
9. Sketch a graph of a function that has these properties.

$f(x)$ is continuous for all real numbers.

$f(x) < 5$ for all real numbers.

$f'(2) = 0$ and $f(2) = 1$

- $f'(x) < 0$ on $(-\infty, 0)$
- $f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$
- $f''(x) < 0$ on $(-\infty, 0)$ and $(0, 2)$ and $(4, \infty)$
- $f''(x) > 0$ on $(2, 4)$



10. The value of a van purchased in 2005 can be approximated by the function $V(x) = 25(0.85)^x$, where x is time in years from 2005, and V is the value in thousand of dollars. Evaluate $V'(4)$ and interpret the result.

$$V'(x) = 25(0.85)^x \cdot \ln(0.85)$$

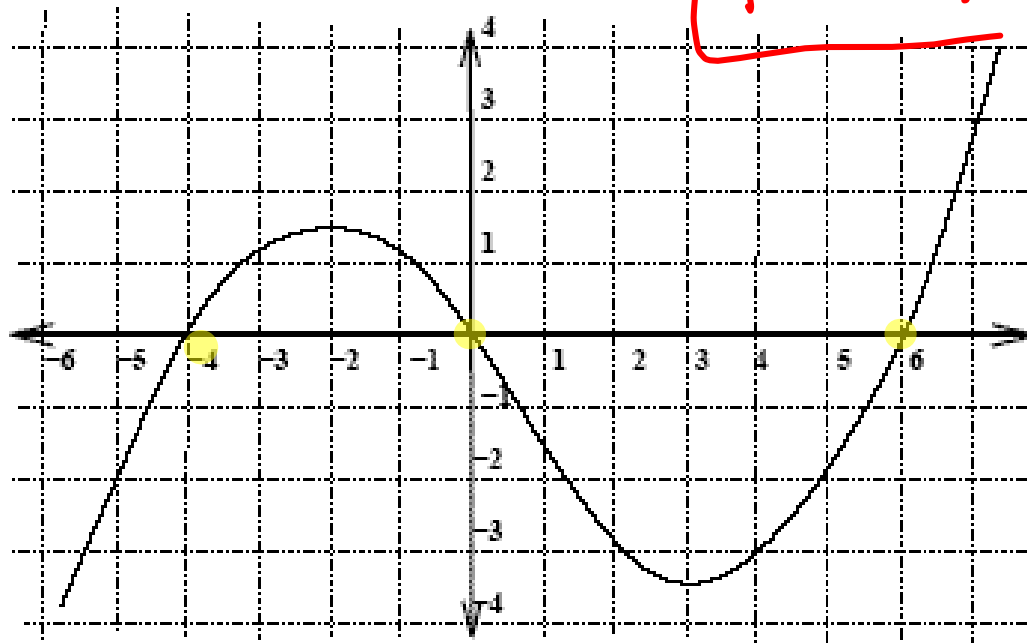
$$V'(4) = \underline{-2.120897} \quad \text{Thousand of \$ per yr}$$

$$\hookrightarrow \$ -2120.90/\text{yr}$$

In 2009 the value of the van will decrease by approximately \$2120.90 in the next yr.

11. Use the graph to answer these questions.

$f'' > 0 \Leftrightarrow f' \text{ inc} \Leftrightarrow f \text{ c.u.}$
 $f'' < 0 \Leftrightarrow f' \text{ dec} \Leftrightarrow f \text{ c.d.}$



(a) If the graph is of $f'(x)$, where is $f(x)$ increasing and concave up?

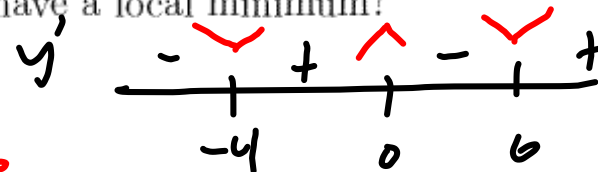
$f'(x) \text{ pos.}$ $f'(x) \text{ inc.}$

$(-4, -2)$, $(6, \infty)$

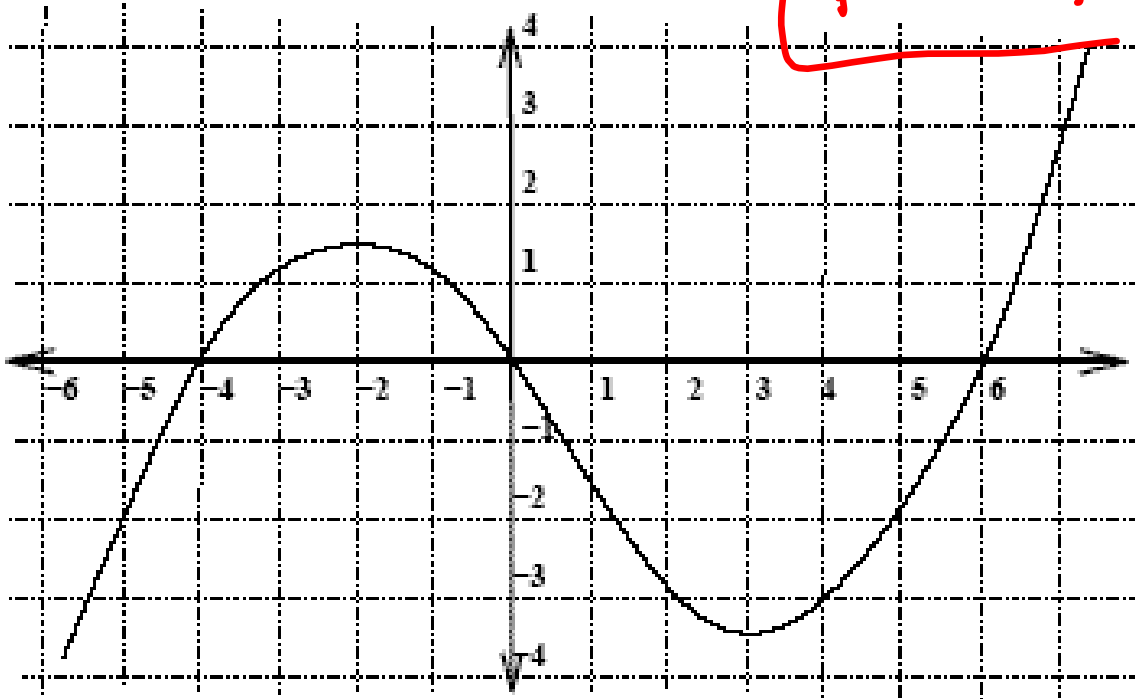
(b) If the graph is of $f'(x)$, where does $f(x)$ have a local minimum?

C.I.V. at $x = -4, 0, 6$

Local min $x = -4, x = 6$



$f'' > 0 \Leftrightarrow f' \text{ inc} \Leftrightarrow f \text{ c.u.}$
 $f'' < 0 \Leftrightarrow f' \text{ dec} \Leftrightarrow f \text{ c.d}$



(c) If the graph is of $f'(x)$, where does $f(x)$ have inflection points?

$x = -2$
 $x = 3$

↳ where $f'(x)$ changes direction

(d) If the graph is of $f''(x)$, where is $f(x)$ concave up? $(-4, 0), (6, \infty)$

12. Find the equation of the tangent line $y = \ln(x^3 - 7) + e^{x^2-4}$ at $x = 2$

$$f(2) = \ln(8-7) + e^{4-4} = \ln(1) + e^0 = 0 + 1 = 1$$

point $(2, 1)$

$$f'(x) = \frac{3x^2}{x^3-7} + 2x e^{x^2-4}$$

$$m_{\text{tan}} = f'(2) = \frac{12}{8-7} + 4 e^{4-4} = 12 + 4 = 16$$

$$y - 1 = 16(x - 2)$$

13. Find $f''(x)$ for $f(x) = e^{x^2+4x}$

$$f' = \underbrace{(2x+4)}_f \underbrace{e^{x^2+4x}}_g$$

$$f'' = \underbrace{2}_{f'} \underbrace{e^{x^2+4x}}_g + \underbrace{(2x+4)}_f \cdot \underbrace{(2x+4)e^{x^2+4x}}_{g'}$$

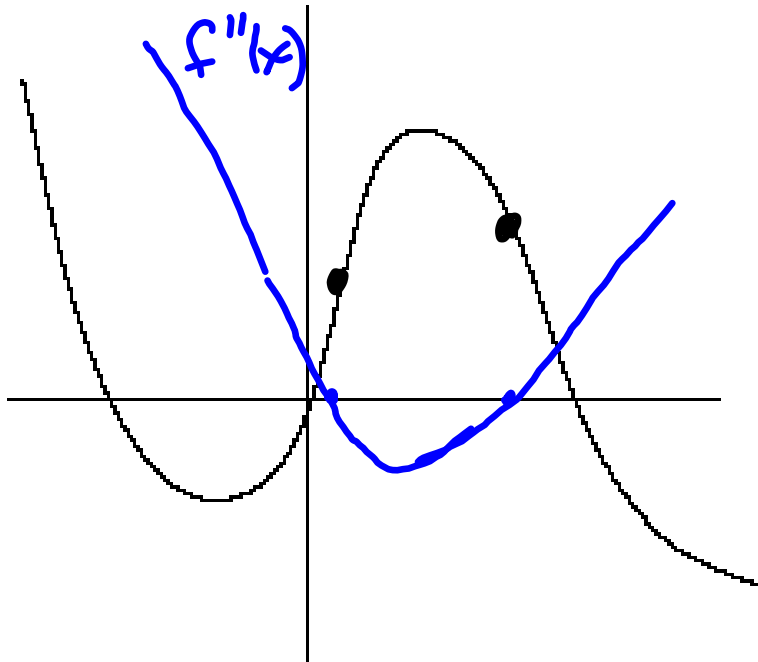
If you simplify.

$$f'' = 2e^{x^2+4x} + (4x^2 + 16x + 16)e^{x^2+4x}$$

$$= [2 + 4x^2 + 16x + 16]e^{x^2+4x}$$

$$= (4x^2 + 16x + 18)e^{x^2+4x}$$

14. Sketch the graph of the 2nd derivative for this function.

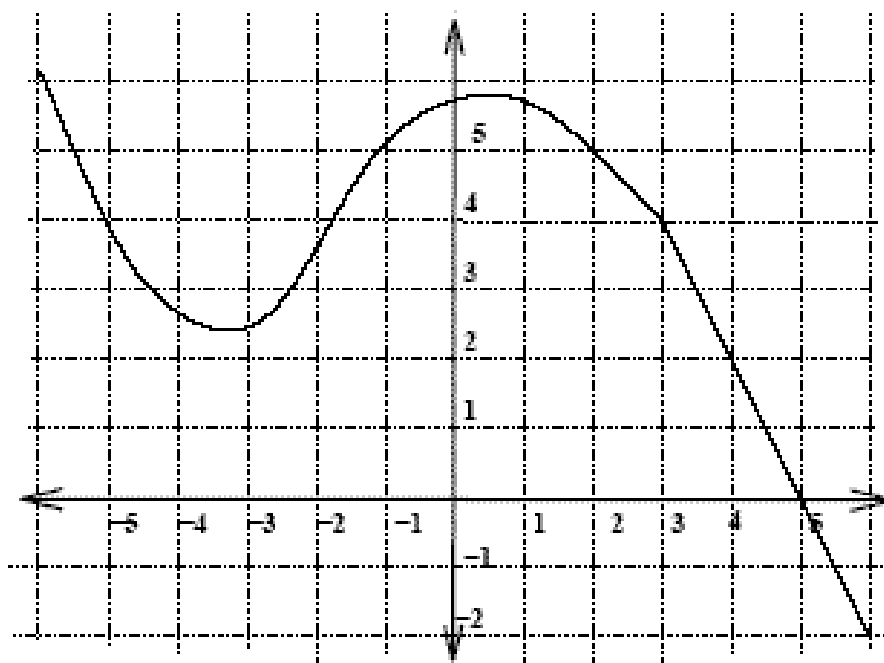


In flexion points
on $f(x)$ are
black dots.

15. Let $f(x)$ and $g(x)$ whose values and derivative values are given by the table. Let $k(x)$ be the function given in the graph. Use this information to compute the following.

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5

$k(x)$



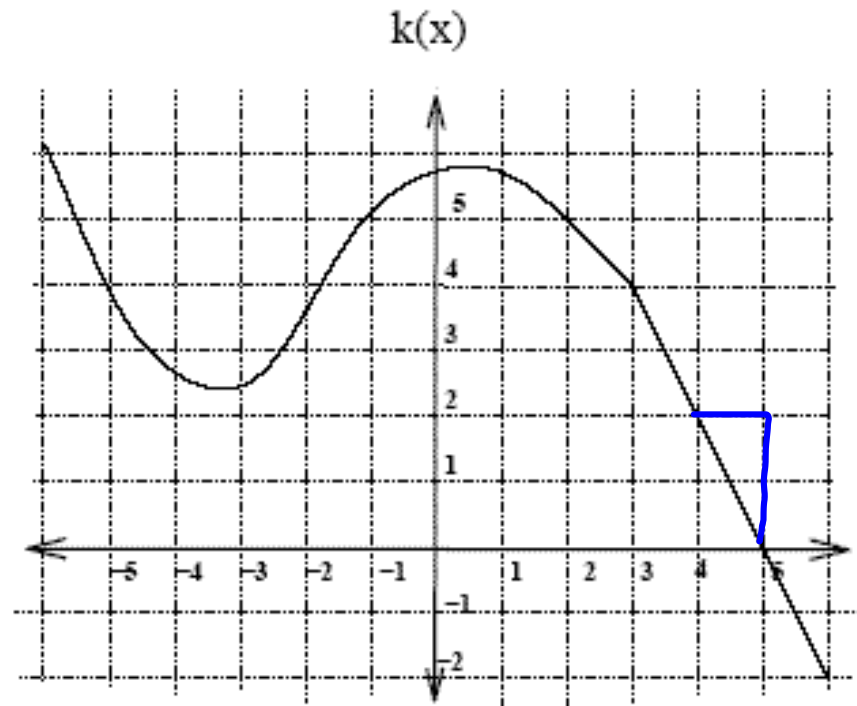
- (a) If $M(x) = \cos(f(x))$, find $M'(0)$

$$M'(x) = -\sin(f(x)) \cdot f'(x)$$

$$M'(0) = -\sin(f(0)) \cdot f'(0)$$

$$= -\sin(1) \cdot 2 = -2\sin(1)$$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5



(b) If $H(x) = x^2 - (k(x))^3$, find $H'(4)$

$$H'(x) = 2x - 3(k(x))^2 \cdot k'(x)$$

$$H'(4) = 8 - 3(k(4))^2 \cdot k'(4)$$

$$= 8 - 3(2)^2 \cdot (-2)$$

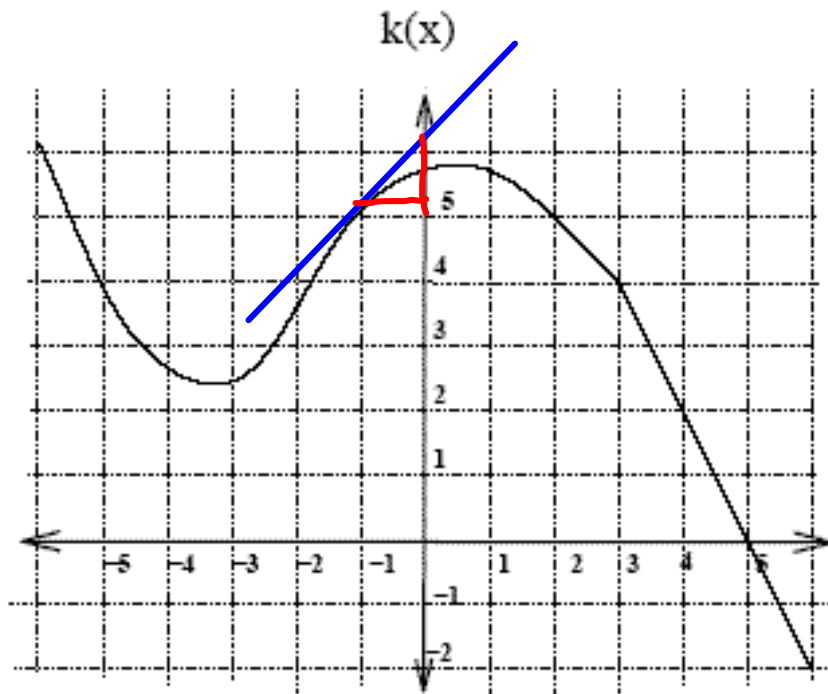
$$= 8 - 3(4)(-2)$$

$$= 8 + 24 = 32$$

$$k(4) = 2$$

$$k'(4) = \frac{-2}{1}$$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5

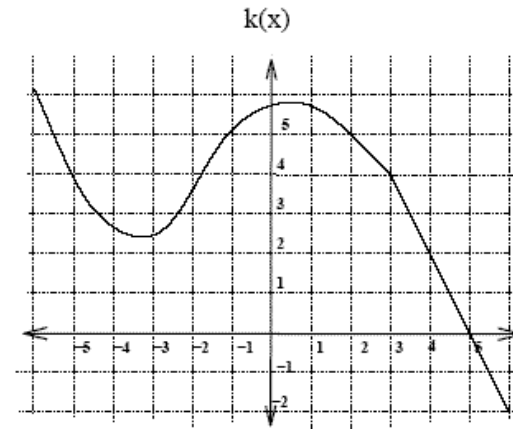


(c) If $J(x) = \underbrace{k(x)} \underbrace{g(x^2)}$, find $J'(-1)$

$$J'(x) = K'(x) g(x^2) + K(x) \cdot g'(x^2) \cdot 2x$$

$$\begin{aligned}
 J'(-1) &= K'(-1) g((-1)^2) + K(-1) \cdot g'((-1)^2) \cdot 2(-1) \\
 &= K'(-1) g(1) + K(-1) \cdot g'(1) (-2) \\
 &= (2)(2) + (5)(3)(-2) \\
 &= 2 + (5)(-6) = \underline{\hspace{2cm}}
 \end{aligned}$$

x	0	1	2
$f(x)$	1	-1	4
$f'(x)$	2	4	6
$g(x)$	-1	2	3
$g'(x)$	5	3	5



(d) If $N(x) = \frac{f(x) + 2x}{g(x)}$, find $N'(2)$

$$N'(x) = \frac{g(x)(f'(x) + 2) - (f(x) + 2x) \cdot g'(x)}{(g(x))^2}$$

$$N'(2) = \frac{g(2)(f'(2) + 2) - (f(2) + 4) \cdot g'(2)}{(g(2))^2}$$

$$= \frac{3(6 + 2) - (4 + 4)(5)}{(3)^2}$$

$$= \frac{3(8) - 8(5)}{9} = \frac{24 - 40}{9}$$

$$= \frac{-16}{9}$$