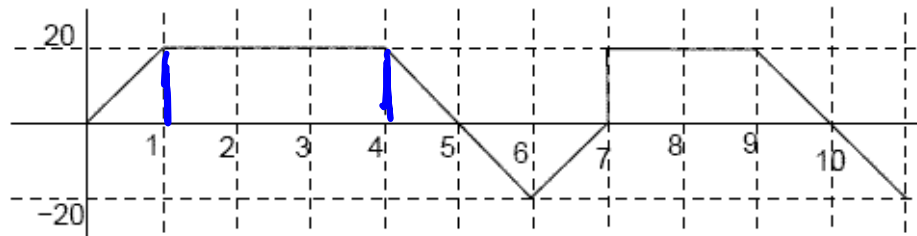


Exam # 3 Sample Review
Sections 5.1-5.5, and 7.1-7.4

1. A bat starts out traveling towards the exit of a tunnel 90 feet away. The graph describes the bat's velocity, $v(x)$ in ft/sec, vs. time.



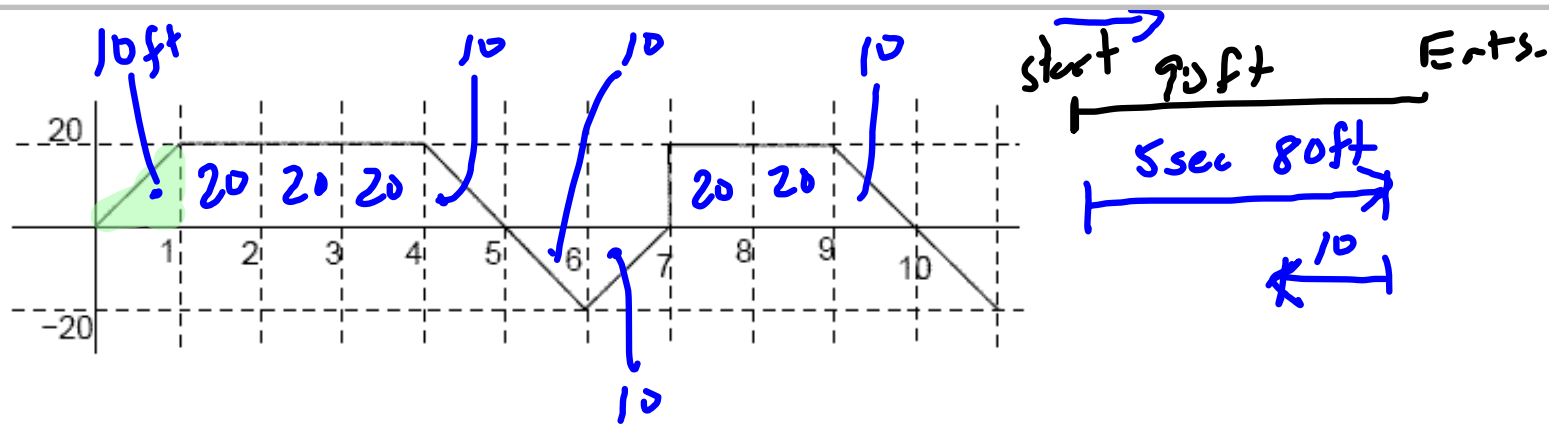
- (a) What is the interpretation of $\int_1^4 v(x) dx$? give the units of the definite integral.

since $v(x) > 0$ for $x=1$ to $x=4$ $\int_1^4 v(x) dx$
is the distance traveled.

$\int_4^7 v(x) dx$ is the total change in the bat's
position from the 4 sec mark to the 7 sec mark.

- (b) When does the bat change direction?

at $x=5, x=7, x=10$



(c) How far from the starting point is the bat after 6 seconds?

$\xrightarrow{80}$
 $\xleftarrow{10}$

70ft from start.

(d) How far has the bat traveled in the 6 seconds?

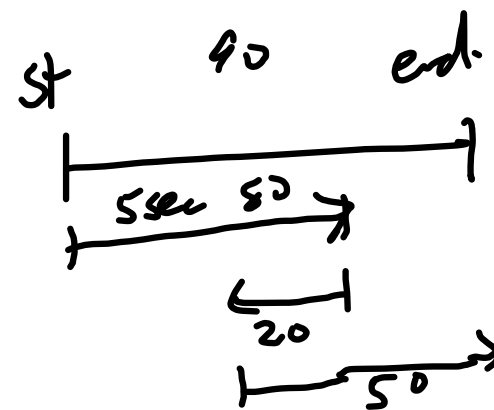
$\xrightarrow{80}$
 $\xleftarrow{10}$

$$\begin{array}{r} 80 \\ + 10 \\ \hline 90 \text{ ft.} \end{array}$$

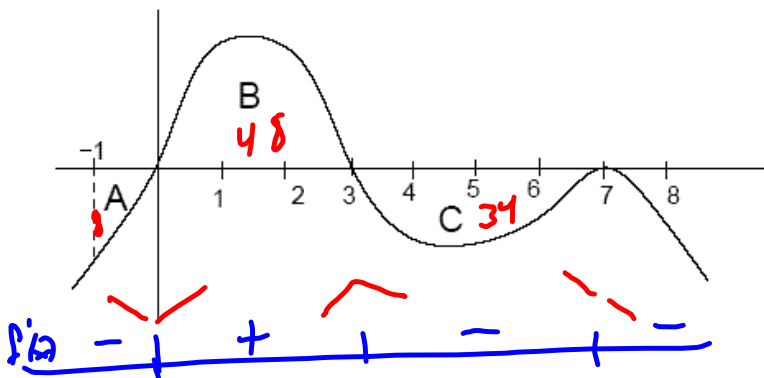
(e) Does the bat make it out of the tunnel in 10 seconds?

$$80 - 20 + 50 = 110 \text{ ft}$$

50 yes



2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A = 8, Region B = 48, and Region C = 34.



- (a) Find the coordinates, (x, y) , for all of the critical values. Also classify the critical values.

C.V. $x = 0, 3, 7$ word.

$x = 0$ is a local min $(0, 40)$

$x = 3$ is a local max $(3, 88)$

$x = 7$ is a neither $(7, 54)$

$$\int_0^3 f'(x) dx = f(3) - f(0)$$

$$48 = f(3) - 40$$

$$88 = f(3)$$

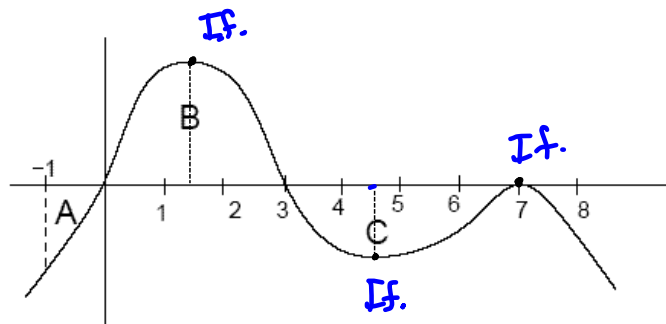
$$\int_3^7 f'(x) dx = f(7) - f(3)$$

$$-34 = f(7) - 88$$

$$54 = f(7)$$

- (b) Find the x-values for the inflection points.
 (c) Sketch the graph of $f(x)$.

2. The following is the graph of $f'(x)$. Use the fact that $f(0) = 40$ along with the given areas for the regions to answer the following. Region A = 8, Region B = 48, and Region C = 34.



- (b) Find the x-values for the inflection points.

$$x \approx 1.5$$

$$x \approx 4.6$$

$$x = 7$$

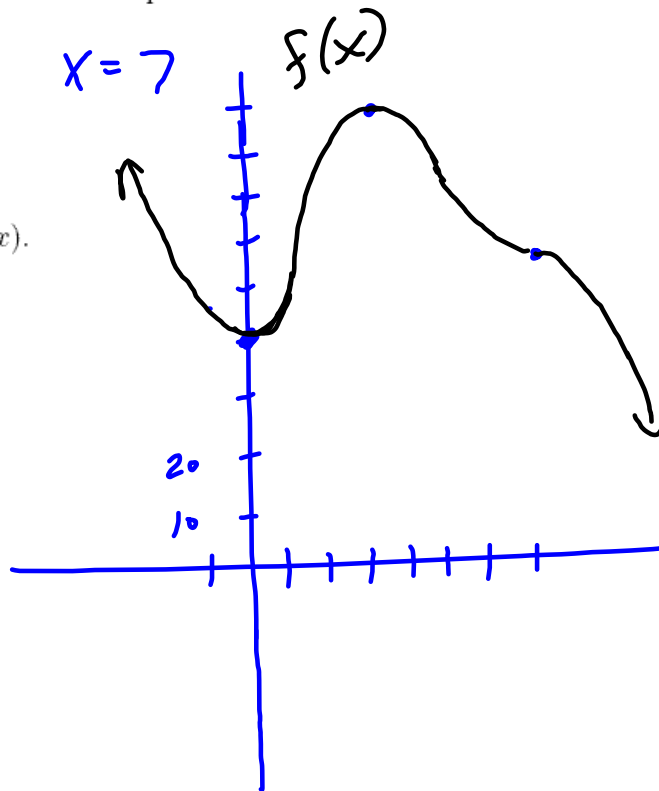
- (c) Sketch the graph of $f(x)$.

$$\int f'(x) = f(x) - f(-1)$$

$$-8 = f(x) - f(-1)$$

$$f(-1) = 40 + 8$$

$$f(-1) = 48$$



3. If x is the number of years from 1990, then the population growth of a city, in millions per year, can be modeled by the formula $1.5e^{0.25x}$.

(a) Write an expression that will give the growth of the city A years after 1990.

$$\int_0^A 1.5e^{.25x} dx = 1.5 \left(\frac{1}{.25}\right) e^{.25x} \Big|_0^A$$

$$= 6e^{.25x} \Big|_0^A = 6e^{.25A} - 6e^0$$

$$= \boxed{6e^{.25A} - 6}$$

(b) If the city had a population of 5 million in 1990, find a formula that will give the population A years after 1990.

$$\int_0^A P'(x) dx = P(A) - P(0)$$

$$6e^{.25A} - 6 = P(A) - 5$$

$$\boxed{6e^{.25A} - 1 = P(A)}$$

4. If $f'(x) = 5 \cos(3x^2)$ and $f(-1) = 2$ then find $f(1)$.

u-sub does not work on this function.

$$\int_{-1}^1 5 \cos(3x^2) dx = f(1) - f(-1)$$

↖ used fInt

$$4.05955 = f(1) - 2$$

$$\boxed{6.05955 = f(1)}$$

5. Compute $\int_J^5 \frac{2}{x^2} + \cos(3x) dx = \int_J^5 2x^{-2} + \cos(3x) dx$

$$= \left. \frac{2x^{-1}}{-1} + \frac{1}{3} \sin(3x) \right|_J^5$$

$$= \left. -\frac{2}{x} + \frac{1}{3} \sin(3x) \right|_J^5$$

$$= \left(-\frac{2}{5} + \frac{1}{3} \sin(15) \right) - \left(-\frac{2}{J} + \frac{1}{3} \sin(3J) \right)$$

6. (a) Compute $\int_0^A \frac{x}{1+x^2} dx = \int_{x=0}^{x=A} \frac{1}{2} \frac{1}{u} du$

$u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x dx$

$\frac{1}{2} du = x dx$

$= \frac{1}{2} \ln|u| \Big|_{x=0}^{x=A}$

$= \frac{1}{2} \ln(1+x^2) \Big|_0^A$

$= \frac{1}{2} \ln(1+A^2) - \frac{1}{2} \ln(1)$

$= \frac{1}{2} \ln(1+A^2)$

(b) Does $\int_0^{\infty} \frac{x}{1+x^2} dx$ converge or diverge?

since $\frac{1}{2} \ln(1+A^2) \rightarrow \infty$ as

$A \rightarrow \infty$

$\int_0^{\infty} \frac{x}{1+x^2} dx$ diverges.

$$\int_1^{\infty} \frac{1}{x^2} dx$$

$$\int_1^A x^{-2} dx = \left. \frac{x^{-1}}{-1} \right|_1^A = \left. -\frac{1}{x} \right|_1^A$$

$$= -\frac{1}{A} - \left(-\frac{1}{1} \right) = -\frac{1}{A} + 1$$

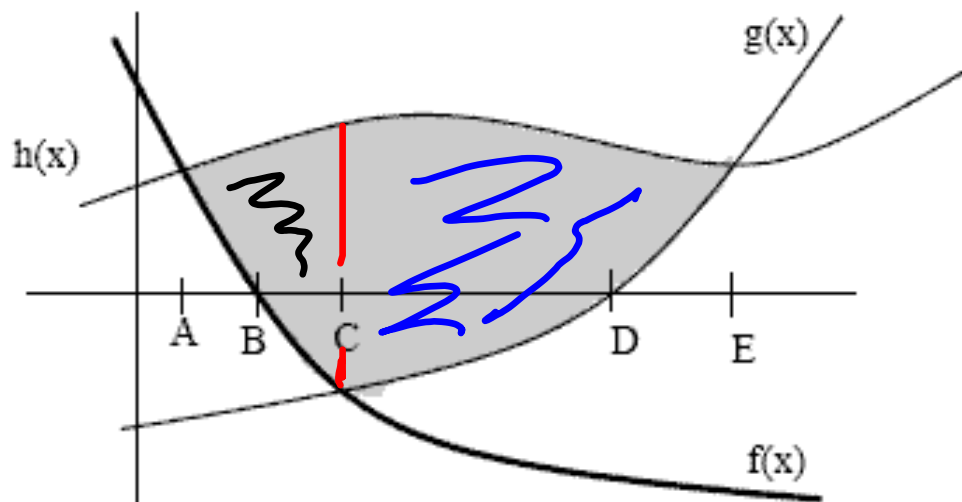
as $A \rightarrow \infty$ (gets very large)

$$-\frac{1}{A} \rightarrow 0$$

Means. $\int_1^{\infty} \frac{1}{x^2} dx = 1$

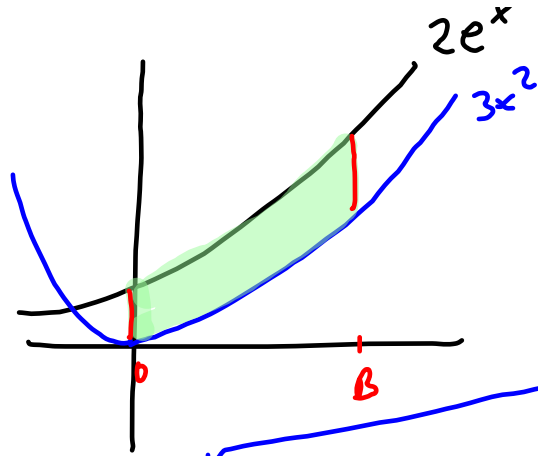
Say integral
converges.

7. Set up the integral(s) that represent this shaded area.



$$\int_A^C h(x) - f(x) dx + \int_C^E h(x) - g(x) dx$$

8. Find the value of B so that the area between the curves $f(x) = 2e^x$ and $g(x) = 3x^2$ from $x = 0$ to $x = B$ will be 110.



$$\int_0^B 2e^x - 3x^2 dx = 110$$

$$\int_0^B 2e^x - 3x^2 dx = e^x - x^3 \Big|_0^B = (2e^B - B^3) - (2e^0 - 0)$$

$$= 2e^B - B^3 - 2$$

$$2e^B - B^3 - 2 = 110$$

$$\underbrace{e^B}_{y_1} - \underbrace{B^3}_{y_2} = 112$$

only way to solve is by graphing on the calc.

$$B = 4.6731$$

9. Compute these integrals

$$(a) \int \frac{1}{2x^3} - \sin(6x) + \sqrt[3]{x} \, dx = \int \frac{1}{2} x^{-3} - \sin(6x) + x^{1/3} \, dx$$

$$= \frac{1}{2} \frac{x^{-2}}{-2} + \frac{1}{6} \cos(6x) + \frac{x^{4/3}}{3 \cdot \frac{4}{3}} + C$$

$$= -\frac{1}{4x^2} + \frac{1}{6} \cos(6x) + \frac{3}{4} x^{4/3} + C$$

$$(b) \int \frac{e^{x^{-1}}}{x^2} dx$$

$$u = x^{-1}$$

$$\frac{du}{dx} = -1x^{-2}$$

$$du = -\frac{1}{x^2} dx$$

$$-du = \frac{1}{x^2} dx$$

$$= \int -e^u du = -e^u + C$$

$$= \boxed{-e^{x^{-1}} + C}$$

$$(c) \int \frac{5e^{5x} - 8e^{-8x}}{(e^{5x} + e^{-8x})^3} dx = \int \frac{1}{u^3} du = \int u^{-3} du$$

$$u = e^{5x} + e^{-8x}$$

$$= \frac{u^{-2}}{-2} + C$$

$$\frac{du}{dx} = 5e^{5x} - 8e^{-8x}$$

$$= -\frac{1}{2} (e^{5x} + e^{-8x})^{-2} + C$$

$$du = (5e^{5x} - 8e^{-8x}) dx$$

or

$$\frac{-1}{2(e^{5x} + e^{-8x})^2} + C$$

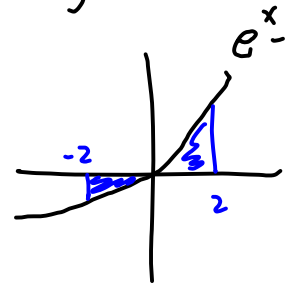
10. Compute the area between the x-axis and $y = e^x - 1$ from $x = -2$ to $x = 2$.

$$\int_{-2}^2 e^x - 1 dx = \text{FnInt}(e^x - 1, x, -2, 2) \\ = 3.25372$$

This is not the answer.

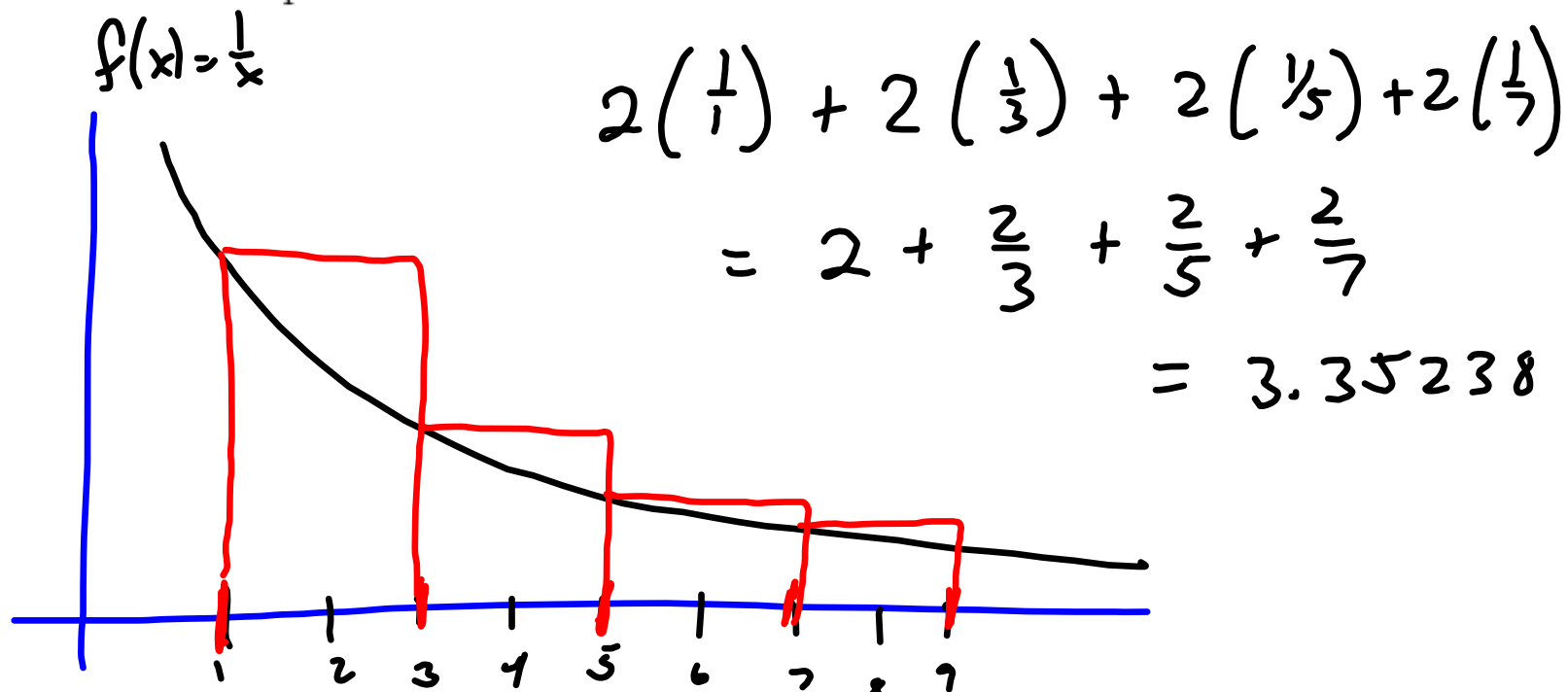
$\int_{-2}^2 e^x - 1 dx$ is the TOTAL Change in the Area.

you need to check the graph first.


$$\left| \int_{-2}^0 e^x - 1 dx \right| + \int_0^2 e^x - 1 dx \\ = |-1.135335| + 4.389056 \\ = 5.524391$$

11. (a) Estimate $\int_1^9 \frac{1}{x} dx$ using a left-hand sum with 4 rectangles.

$$\begin{aligned} \text{base} &= \frac{9-1}{4} \\ &= \frac{8}{4} = 2 \end{aligned}$$



- (b) Is the estimate an over estimate or an under estimate? Justify your answer with a graph.

over since rect.
go above graph of $\frac{1}{x}$

12. Estimate $\int_1^{10} f(x)dx$ using a right sum and the information in the table.

x	1	4	8	10	11
f(x)	5	20	16	8	3

$$3(20) + 4(16) + 2(8)$$

if you need graph the points

