Definition: $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ is a vector function, then the derivative of $\mathbf{r}(t)$ is given by

$$\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle = x'(t)\mathbf{i} + y'(t)\mathbf{j}$$

provided both $x'(t)$ and $y'(t)$ exist.

acceleration

$r''(t) = a(t)$

When $\mathbf{r}(t)$ represents a position function, then $\mathbf{r}'(t)$ is velocity and $|\mathbf{r}'(t)|$ is speed.

$r'(t) = v(t)$ speed $= |v(t)|$

Example: Assume that $\mathbf{r}(t)$ is a position function for an object. Find the velocity vector(s) and the speed at the point $(3, 0)$ when

$\mathbf{r}(t) = \langle t^2 - 6t + 8, t^4 - 26t^2 + 25 \rangle$

$x = 3$
$y = 0$

find value(s) of t

$x = t^2 - 6t + 8$
$3 = t^2 - 6t + 8$
$0 = t^2 - 6t + 5$
$(t-5)(t-1)$
$t = 5 \quad t = 1$

$y = t^4 - 26t^2 + 25$
$0 = t^4 - 26t^2 + 25$

$t = 1$ ✓
$1^4 - 26(1)^2 + 25 =$
$1 - 26 + 25 = 0$

$t = 5$ ✓
$5^4 - 26 \cdot 5^2 + 25$
$625 - 26 \cdot 25 + 25$
$650 - 26 \cdot 25 = 0$

$v(t) = r'(t) = \langle 2t - 6, 4t^3 - 52t \rangle$

$t = 1$

$V(1) = r'(1) = \langle 2 - 6, 4 - 52 \rangle$
$= \langle -4, -48 \rangle$

Speed $= |v(1)| = \sqrt{16 + (-48)^2}$

$t = 5$

$V(5) = r'(5) = \langle 10 - 6, 4(5)^3 - 52(5) \rangle$
$= \langle 4, 240 \rangle$

Speed $= |V(5)| = \sqrt{16 + (240)^2}$

Example: Find the derivative of $\mathbf{r}(t) = \left\langle \frac{t}{\tan(t)}, \cos(3t^2) \right\rangle$. $= \langle t \cot(t), \cos(3t^2) \rangle$

$r'(t) = \langle 1 \cot(t) + t(-\csc^2 t), -\sin(3t^2) \cdot 6t \rangle$

$\langle \cot(t) - t\csc^2(t), -6t \sin(3t^2) \rangle$

Example: For the vector function, $\mathbf{r}(t) = \langle 10t^2, 5t^3 + 7\rangle$, find a tangent vector of unit length when $t = 2$.

$$r'(t) = \langle 20t, 15t^2 \rangle$$

$$r'(2) = \langle 40, 60 \rangle$$ Tangent vector.

$$|r'(2)| = \sqrt{(40)^2 + (60)^2}$$
$$= \sqrt{1600 + 3600}$$
$$= \sqrt{5200}$$
$$= 10\sqrt{52}$$

untit tangent vector $\left\langle \frac{40}{10\sqrt{52}}, \frac{60}{10\sqrt{52}} \right\rangle = \left\langle \frac{4}{\sqrt{52}}, \frac{6}{\sqrt{52}} \right\rangle$

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find a tangent line equation at $t=2$

point
$$r(2) = \langle 40, 47 \rangle$$

$$r(t) = r_0 + tv$$
$$= \langle 40, 47 \rangle + t\langle 40, 60 \rangle$$

$$r(t) = \langle 40, 47 \rangle + t\left\langle \frac{4}{\sqrt{52}}, \frac{6}{\sqrt{52}} \right\rangle$$