

Section 1.5: Inverse Trigonometric Functions

Definition: A function is a rule that assigns to each element in set A exactly one element in set B. Set A is called the **domain**. The **range** of f is the set of all possible values of $f(x)$ where x is in the domain, i.e. $\text{range} = \{f(x) | x \in A\}$.

Example: Find the domain of $f(x) = \frac{x}{x^2 - 25}$

Domain is all Reals except $x = \pm 5$

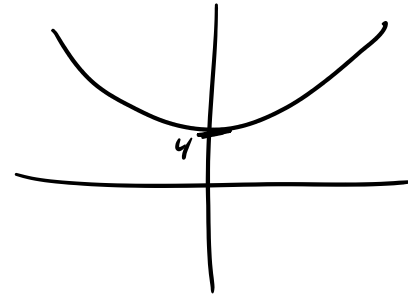
$$(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$$

Definition: A function is said to be **one-to-one** if it never takes on the same function value more than once. i.e. if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.

Definition: Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A and is defined, for any y in B, by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

$$y = x^2 + 4$$

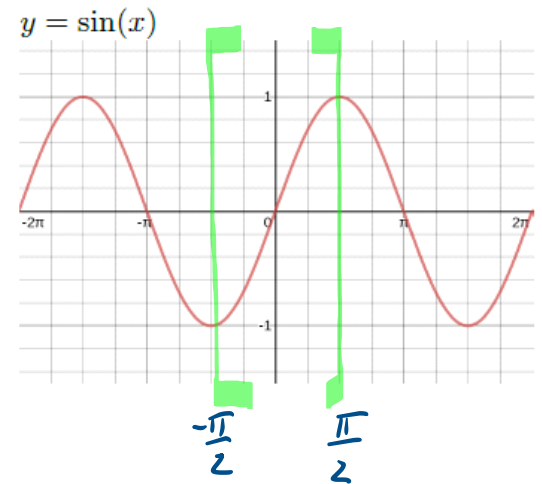


$$y = \arcsin(x) = \sin^{-1}(x)$$

$$-1 \leq x \leq 1$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

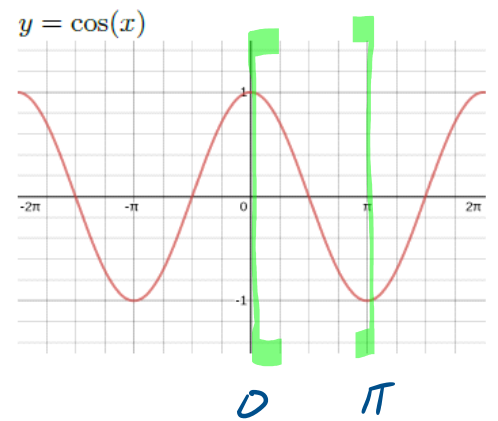
$$\sin y = x$$



$$y = \arccos(x) = \cos^{-1}(x)$$

$$-1 \leq x \leq 1$$

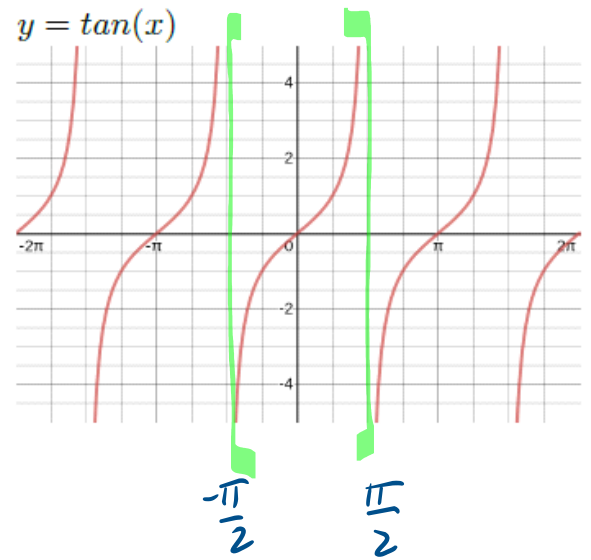
$$0 \leq y \leq \pi$$



$$y = \arctan(x) = \tan^{-1}(x)$$

x any Real #.

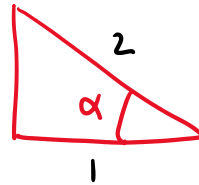
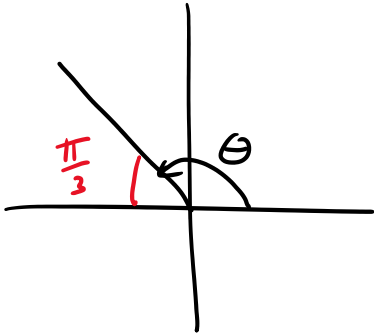
$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$



Example: Find the exact value of the expression.

$$A) \cos^{-1}\left(\frac{-1}{2}\right) = \theta \rightarrow \cos\theta = \frac{-1}{2}$$

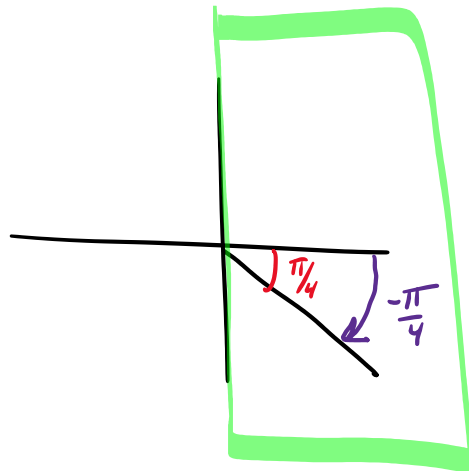
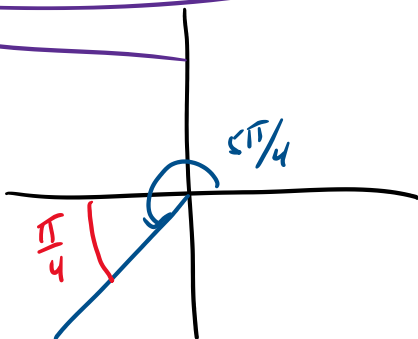
$$\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$$



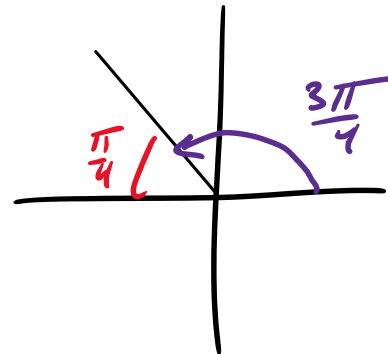
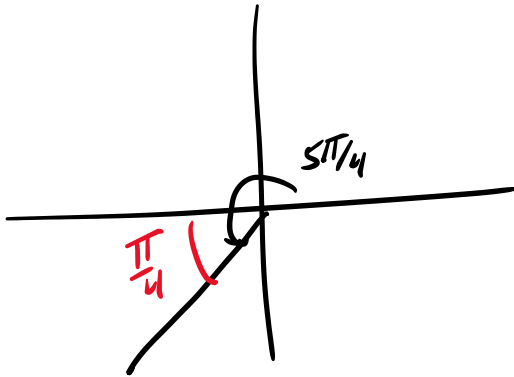
$$\cos\alpha = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3}$$

$$B) \arcsin\left(\sin\frac{5\pi}{4}\right) = -\frac{\pi}{4}$$



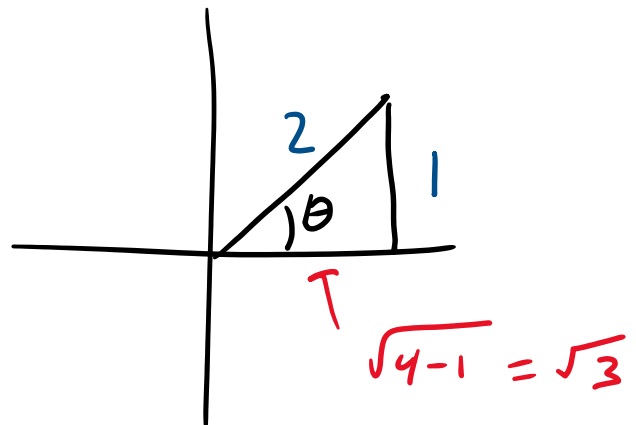
$$C) \arccos\left(\cos\frac{5\pi}{4}\right) = \frac{3\pi}{4}$$



$$D) \cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \cos\theta = \frac{\sqrt{3}}{2}$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \theta$$

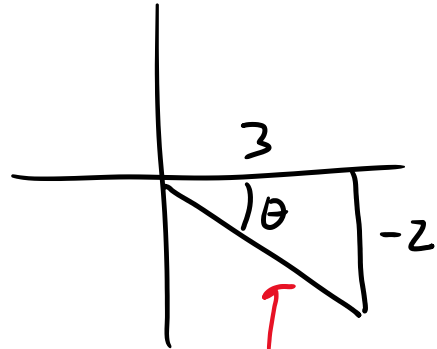
$$\sin\theta = \frac{1}{2}$$



$$E) \sec\left(\tan^{-1}\left(\frac{-2}{3}\right)\right) = \sec \theta = \frac{\sqrt{13}}{3}$$

$$\tan^{-1}\left(\frac{-2}{3}\right) = \theta$$

$$\tan \theta = \frac{-2}{3}$$

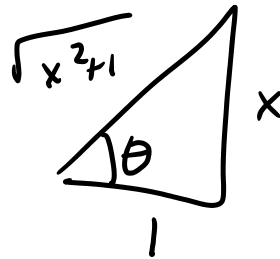


$$\sqrt{9+4} = \sqrt{13}$$

$$F) \sin(\tan^{-1}(x)) = \sin(\theta) = \frac{x}{\sqrt{x^2+1}}$$

$$\tan^{-1}(x) = \theta$$

$$\tan \theta = \frac{x}{1}$$



$$G) \cos\left(\sin^{-1}\left(\frac{x}{5}\right)\right) = \cos \theta = \frac{\sqrt{25-x^2}}{5}$$

$$\sin^{-1}\left(\frac{x}{5}\right) = \theta$$

$$\sin \theta = \frac{x}{5}$$

