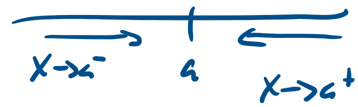
Section 2.2: The Limit of a Function

A limit is way to discuss how the values of a function(y-values) are behaving when x gets close to the number a. There are three forms to the limit.



$\underbrace{\lim_{x \to a^-}}$ $f(x)$ Left

$\underbrace{\lim_{x \to a^+}}$ $f(x)$ Right.

$\underbrace{\lim_{x \to a}}$ $f(x)$ Two direction Limit

We write $\lim_{x \to a^-} f(x) = L$ and say "the limit of $f(x)$ as x approaches a from the left, equals L"

$\lim_{x \to a} f(x) = L$ if and only if

$\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

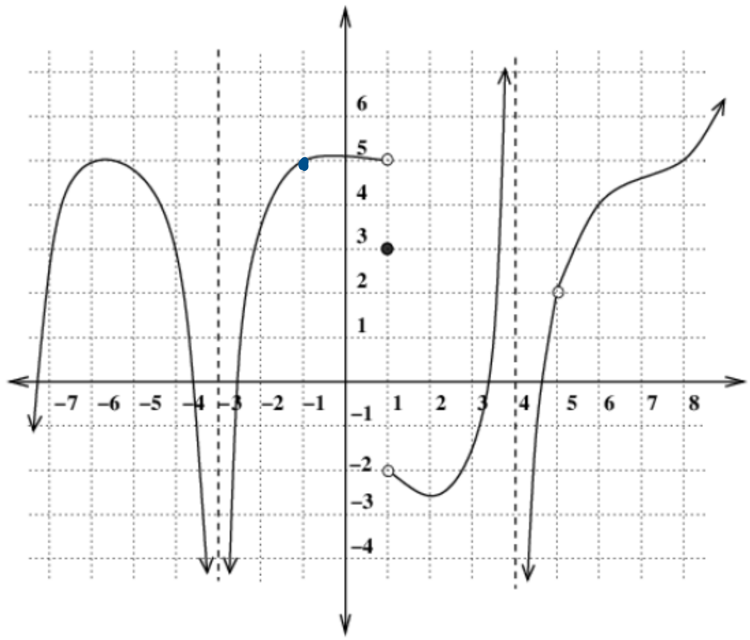
Evaluating Limits Graphically

Example: Use the graph to answer the following questions.

$\lim_{x \to -1^-} f(x) =$ 5

$\lim_{x \to -1^+} f(x) =$ 5

$\lim_{x \to -1} f(x) =$ 5

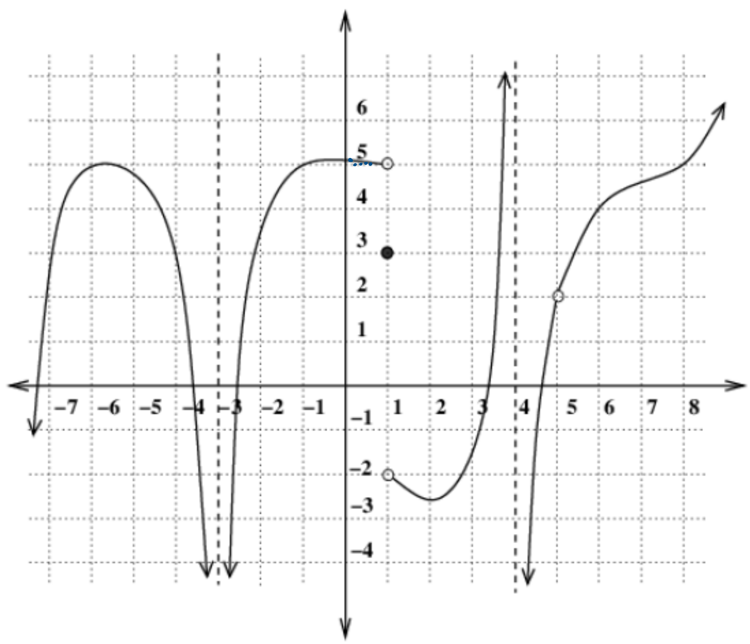


$\lim_{x \to 1^-} f(x) =$ 5

$\lim_{x \to 1^+} f(x) =$ −2

$\lim_{x \to 1} f(x) =$ DNE

$f(1) =$ 3

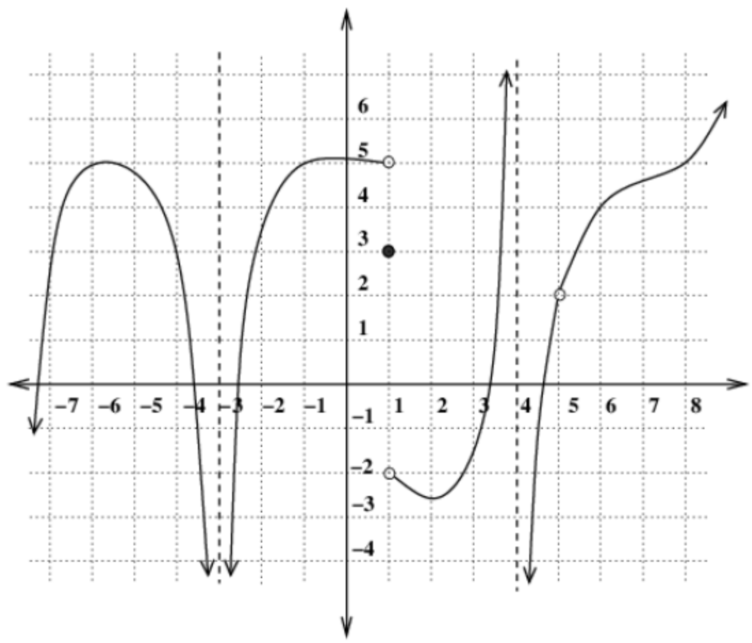


$\lim_{x \to 5^-} f(x) =$ 2

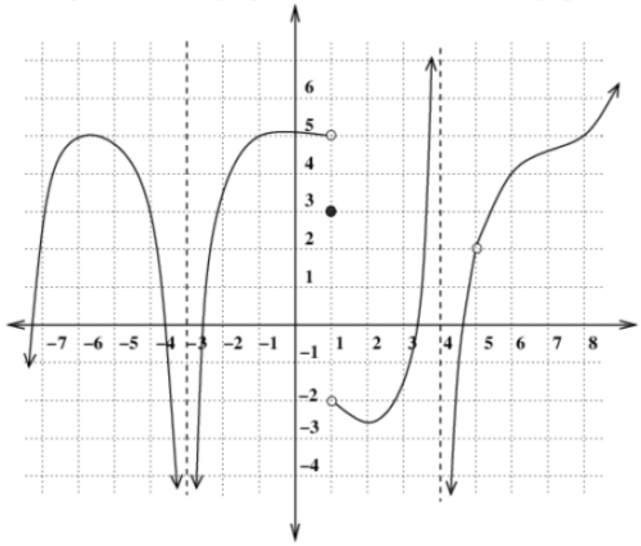
$\lim_{x \to 5^+} f(x) =$ 2

$\lim_{x \to 5} f(x) =$ 2

$f(5) =$ DNE



Example: Use the graph to answer the following questions.



$\lim_{x \to -3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = -\infty$

$\lim_{x \to -3} f(x) = -\infty$

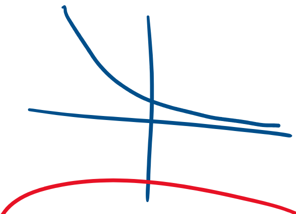
$\lim_{x \to 4^-} f(x) = +\infty$

$\lim_{x \to 4^+} f(x) = -\infty$

$\lim_{x \to 4} f(x) =$ DNE

$\underbrace{\lim_{x \to \infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = -\infty}_{\text{end Behavior of the function}}$

$y = e^{-x}$



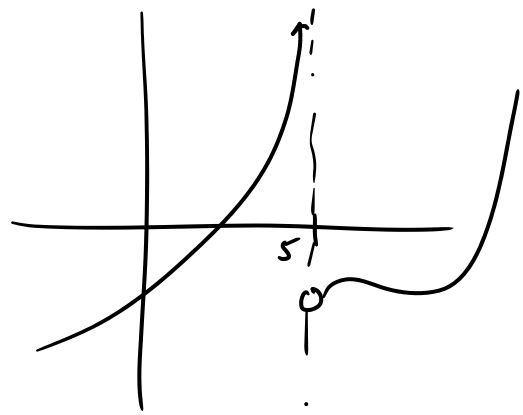
X=-3 and x=4 are vertical asymptotes

Definition: $\underbrace{x = a}$ is said to be a **vertical asymptote** of the function $f(x)$ provided that at least one of the following statements is true:

$$\lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

$x=5$ V.A.



Evaluating Limits with Tables

Example: Compute the limit.

$$\lim_{x\to 0} \frac{\sqrt{x^2+16}-4}{x^2} = .125$$

x	$f(x)$
1	0.1231056
0.5	0.124515
0.1	0.1249804
0.05	0.1249951
0.001	0.1249998

x	$f(x)$
-1	0.1231056
-0.5	0.124515
-0.1	0.1249804
-0.05	0.1249951
-0.001	0.1249998

$\lim_{x\to 0^+} f(x) = .125$

$\lim_{x\to 0^-} f(x) = .125$

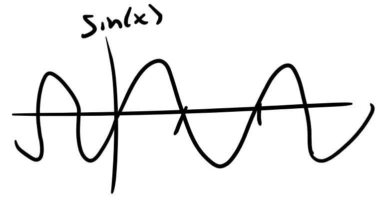
Example: Compute the limit.

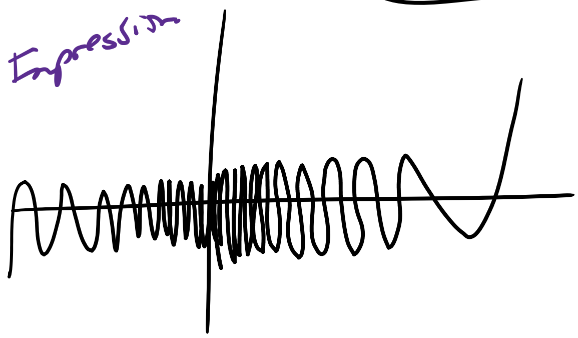
$$\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right) = \text{DNE}$$

x	$f(x)$
1	0
$\frac{1}{2}$	0
$\frac{1}{3}$	0
$\frac{1}{4}$	0

Correct values
But
wrong Impression

$$\sin\frac{\pi}{\frac{1}{2}} = \sin(2\pi)$$





Example: Evaluate these limits.

A) $\lim_{x\to 4^+} \frac{1}{x-4} =$ $+\infty$

$f(x) = \frac{1}{x-4}$

x	$f(x)$
4.1	10
4.01	100
4.001	1000
4.0001	10000

$\frac{1}{.1} = \frac{10}{1}$

$\frac{1}{.01} = \frac{100}{1}$

$x=4$ not in the domain.

B) $\lim_{x\to 0} \frac{1}{x^2} =$ $+\infty$

C) $\lim_{x\to 0} \frac{1}{x^3} =$ DNE

$\lim_{x\to 0^+} \frac{1}{x^3} = +\infty$ $\qquad$ $\lim_{x\to 0^-} \frac{1}{x^3} = -\infty$