Section 3.7: Rates of Change in the Natural and Social Sciences

Example: An object is moving in a straight line. Its position is given by $s(t) = 4t^3 - 9t^2 + 6t + 2$, where t is measured in seconds and s is measured in meters.

A) Find the velocity of the object at time t.

$$v(t) = s'(t) = 12t^2 - 18t + 6$$

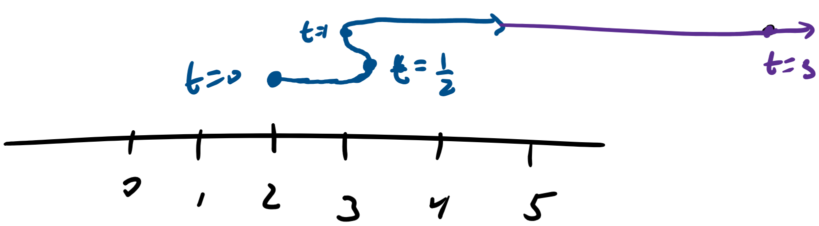
B) When is the object at rest? → $v(t) = 0$

$$0 = 12t^2 - 18t + 6$$
$$= 6(2t^2 - 3t + 1)$$
$$= 6(2t-1)(t-1)$$
$$t = \frac{1}{2} \qquad t = 1$$

$v(t) = 6(2t-1)(t-1)$

$v(t)$: $+$ | $\frac{1}{2}$ | $-$ | 1 | $+$

C) Draw a diagram to represent the motion of the object.



$s(0) = 2$

$s(\frac{1}{2}) = 3.25$

$s(1) = 3$

D) Find the total distance the object traveled during the first three seconds.

$s(3) = 47$

$s(3) - s(0)$ ← displacement.

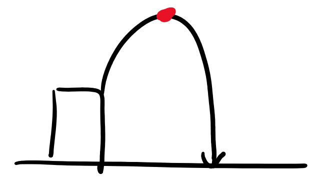
$= 47 - 2 = 45\,m$

$s(\frac{1}{2}) - s(0) = 3.25 - 2 = 1.25$

$s(\frac{1}{2}) - s(1) = 3.25 - 3 = .25$

$s(3) - s(1) = 47 - 3 = 44$

total $= 45.5\,m$



Example: The height in meters of a rocket launched vertically upward from a point 2m above the ground is $h = 2 + 24.5t - 4.9t^2$ after t seconds.

A) Find the position and velocity after 4 seconds.

$v = h' = 24.5 - 9.8t$

$h(4) = 2 + 24.5(4) - 4.9(4)^2 = 21.6m$

$v(4) = 24.5 - 9.8(4) = -14.7 m/sec.$

B) When does the rocket reach its maximum height? What is the maximum height?

$v(t) = 0$

$24.5 - 9.8t = 0$

$t = \frac{24.5}{9.8}$ sec. $= 2.5$sec

$h(2.5) = 32.625 m.$

C) When does the rocket hit the ground.?

$h(t) = 0$

$2 + 24.5t - 4.9t^2 = 0$

$t = \frac{-24.5 \pm \sqrt{24.5^2 - 4(-4.9)(2)}}{2(-4.9)}$

$t = -.0803$ $\quad$ $t = 5.0803$

D) What is the velocity and the speed when the rocket hits the ground?

$v(5.0803) = -25.29 m/sec.$

speed $= |v(5.0803)| = 25.29 m/sec.$

Example: A stone is dropped into a still pond creating a ripple that travels outward at a speed of 40 cm/sec. Find the rate at which the area within the circle is increasing after 3 seconds.

find $A'(3)$

when $t = 3$ sec.

$\frac{d}{dt} A(t)$

$A = \pi r^2$

$r = 40t$

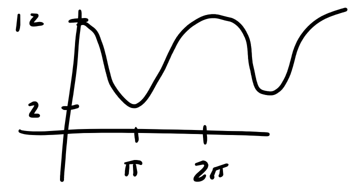
$A(t) = \pi(40t)^2$

$A(t) = 1600\pi t^2$

$A' = 3200\pi t$

$A'(3) = 3200\pi(3) = 9600\pi \text{ cm}^2/\text{sec.}$

$7 + 5\cos\theta$



Example: The tides at a particular location can be modeled by the formula $D(t) = 7 + 5\cos[0.503(t - 6.5)]$ where t is the time in hours after midnight on the day of the start of the model.

depth is in meters.

A) What time was the first high tide after the start of the model? Low tide?

High tide: need $.503(t-6.5) = 0$

$t = 6.5$

Time: 6:30 Am

$.503(t-6.5) = -2\pi$

$t - 6.5 = \frac{-2\pi}{.503}$

$t = 6.5 + \frac{-2\pi}{.503}$

at $\theta = -2\pi$

$t =$ neg.

Low tide: need $.503(t-6.5) = \pi$

$t - 6.5 = \frac{\pi}{.503}$

$t = 6.5 + \frac{\pi}{.503} \simeq 12.75$ hrs

Time: 12:45 pm

B) What is the rate of change of the tide at 9AM? at 2PM?

$D(t) = 7 + 5\cos(.503(t-6.5))$

$D'(t) = -5\sin(.503(t-6.5)) \cdot (.503)$

at 9Am: $D'(9) = -2.353$ m/hr.

at 2pm: $D'(14) = 1.4835$ m/hr.