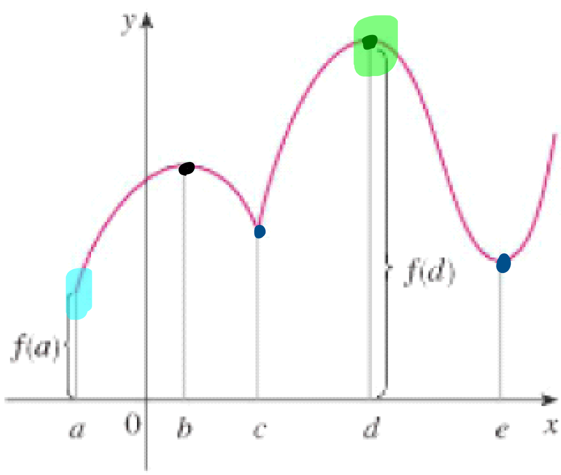
Sections 4.1-4.3 Part 3: Absolute Maximum/Minimum and other Theorems

Absolute Maxima and Minima

Definition: Let c be a number in the domain of a function f. Then $f(c)$ is the

- **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in the domain.
- **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in the domain.



Abs min $f(a)$

abs max $f(d)$

loc min $f(c), f(e)$

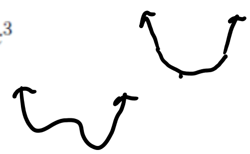
loc max $f(b), f(d)$

Example: Find the absolute max and the absolute min.

A) $y = x^3 + 3x^2 + 1$

no Abs max / Abs min

B) $y = x^4 - 4x^3$



no Abs max.

Abs min is at

$X = 3$

Abs min is $y(3) = \underline{\underline{-27}}$

$y' = 4x^3 - 12x^2 = 4x^2(x-3)$

$0 = 4x^2(x-3)$

cv $x=0$ $x=3$

y': $-$ | $-$ | $+$

-1, 0, 1, 3, 5

C) $y = 7 + 3\sin(x+10)$

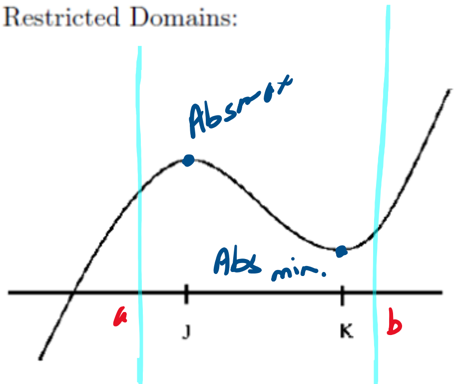
Abs max $= 10$

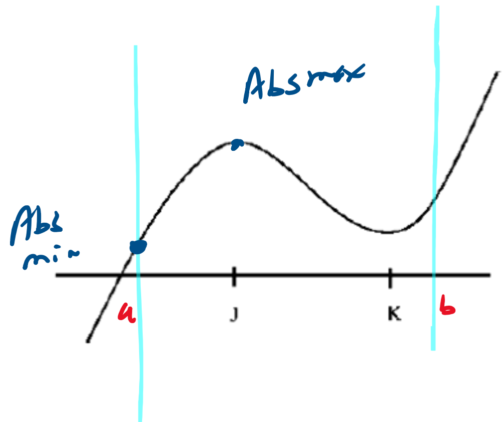
Abs min $= 4$

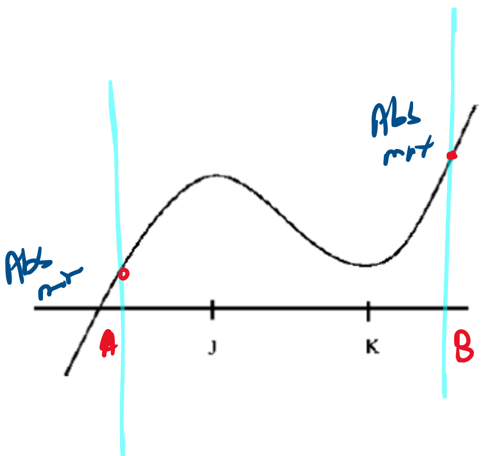
$a \le x \le b$

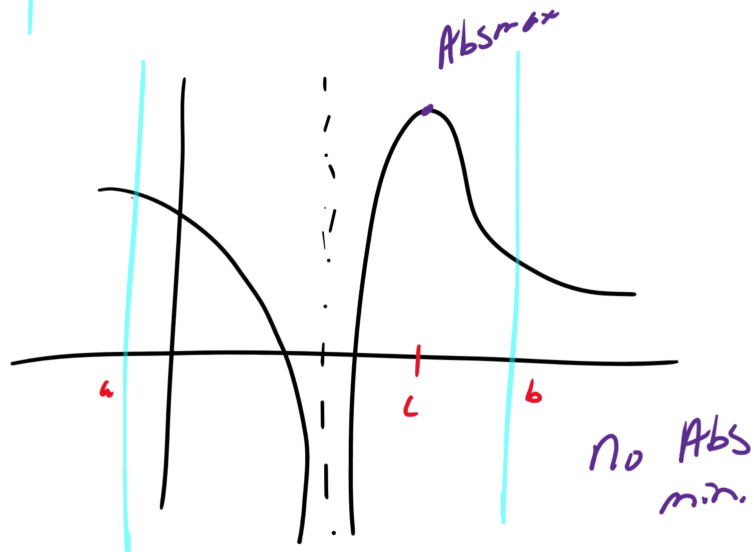
The Extreme Value Theorem: If f is a continuous on a <u>closed interval</u> $[a, b]$, then f will have both an absolute max and an absolute min. They will happen at either critical values in the interval or at the ends of the interval, $x = a$ or $x = b$.

Restricted Domains:









Example: For the function, find the absolute max and the absolute min on the indicated interval.

$f(x) = 12x^2 - 2x^3 + 1$ $\qquad$ $f'(x) = 24x - 6x^2 = 6x(4-x)$ → C.V. $x=0$ $\quad x=4$

f(x) cont.

A) $[2, 5]$

Closed Interval

Zero is not in it :)

$f(2) = 33$ Abs min

$f(4) = 65$ Abs max

$f(5) = 51$

B) $[-3, 5]$

Closed Interval

$f(-3) = 163$ Abs max

$f(0) = 1$ Abs min

$f(4) = 65$

$f(5) = 51$

C) $(-3, 5]$

not a closed Interval

$-3 < x \le 5$

open dot

$f(-3) = 163$ open dot

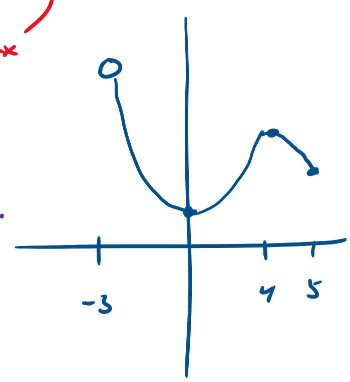
no Abs max

$f(0) = 1$

$f(4) = 65$

$f(5) = 51$

Abs min.



Example: For the function, find the absolute max and the absolute min on the interval $[0, 5]$.

$f(x) = \frac{1}{(x-4)^2}$ $\Leftarrow$ not continuous at $x=4$ $\longrightarrow$ VA @ $x=4$

$f(x) = (x-4)^{-2}$

$f'(x) = -2(x-4)^{-3}$

$f'(x) = \frac{-2}{(x-4)^3}$

$\underline{f'(x) \text{ DNE}}$ $x=4$ not a c.v.

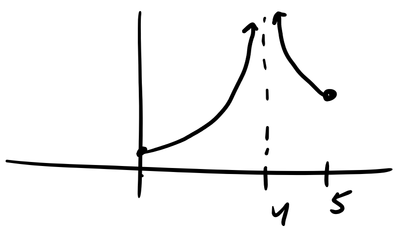
$\underline{f'(x)=0}$ none.

$\lim_{x \to 4^-} \frac{1}{(x-4)^2} = +\infty$

$\lim_{x \to 4^+} \frac{1}{(x-4)^2} = +\infty$

$f(0) = \frac{1}{16}$

$f(5) = 1$



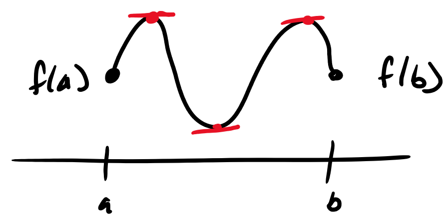
no Abs max

Abs min is $\frac{1}{16}$

Rolle's Theorem: Let f be a function that satisfies the following three hypotheses:

1) f is continuous on the closed interval $[a, b]$.
2) f is differentiable on the open interval (a, b).
3) $f(a) = f(b)$

Then there is a number c between a and b such that $f'(c) = 0$.



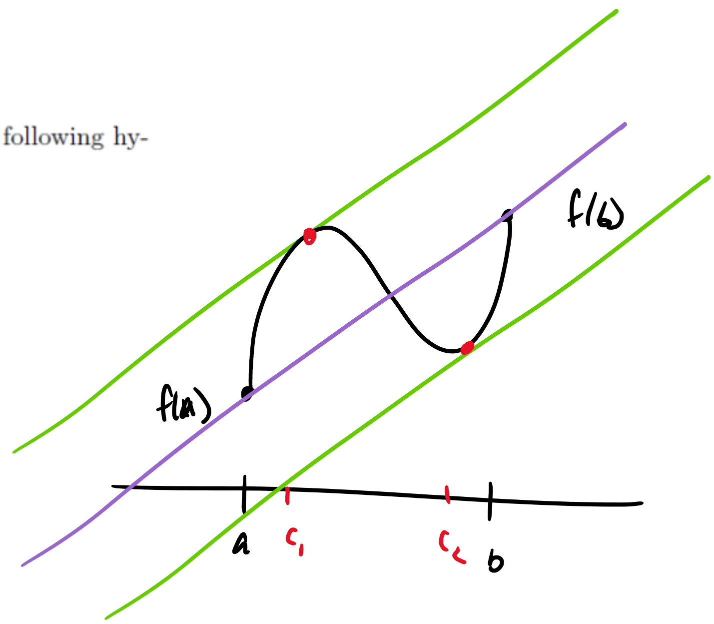
The Mean Value Theorem: Let f be a function that satisfies the following hypotheses:

1) f is continuous on the closed interval $[a, b]$.
2) f is differentiable on the open interval (a, b).

Then there is a number c between a and b such that

$$f'(c) = \underbrace{\frac{f(b) - f(a)}{b - a}}$$

The slope of the line through $(a, f(a))$ & $(b, f(b))$



Example: Find a number c that satisfies the conclusion of the Mean Value Theorem on the interval $[0, 2]$.

$f(x) = x^3 + x - 1$ ← continuous + diff.

$f(2) = 8 + 2 - 1 = 9$

$f(0) = -1$

$f'(x) = 3x^2 + 1$

$f'(c) = 3c^2 + 1$

by MVT there is some value c such that

$0 < c < 2$ and

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$3c^2 + 1 = \frac{9 - -1}{2 - 0}$$

$$3c^2 + 1 = \frac{10}{2} = 5$$

$$3c^2 + 1 = 5$$

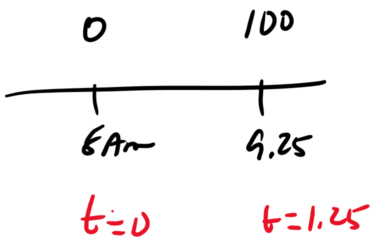
$$3c^2 = 4$$

$$c^2 = \frac{4}{3} \rightarrow c = \pm\sqrt{\frac{4}{3}}$$

$$c = \sqrt{\frac{4}{3}}$$

$$c = -\sqrt{\frac{4}{3}}$$

Example: You enter a toll road at 8am and then exit it at 9:15am. The distance between the entrance and exit is 100 miles. If the maximum speed is set at 70mph, do you get charged for speeding?



position is cont., diff.

by mean Value thm there is a c value $0 < c < 1.25$

such that $f'(c) = \frac{f(1.25) - f(0)}{1.25 - 0}$

$f'(c) = \frac{100}{1.25} = 80$ mph.