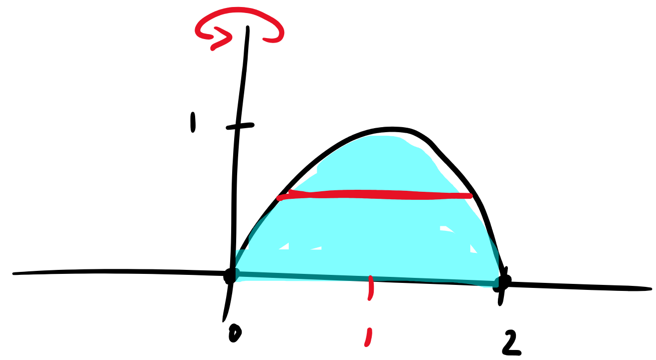
Example: Find the volume of the solid obtained by rotating the region bounded by the given curves around the y-axis.

$y = 2x - x^2 = x(2-x) = 0$

x–axis

washer.



$r_o = 1 + \sqrt{1-y}$

$r_i = 1 - \sqrt{1-y}$

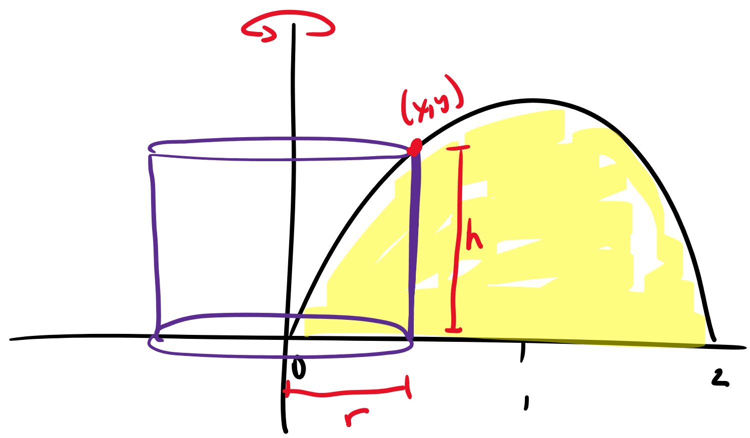
$$\int_0^1 \pi \left[\left(1 + \sqrt{1-y}\right)^2 - \left(1 - \sqrt{1-y}\right)^2 \right] dy$$

Page 1: method 2

Example: Find the volume of the solid obtained by rotating the region bounded by the given curves around the y-axis.

$y = 2x - x^2$ $= (2-x)x$
$x-$axis

shell method.
slice parallel to axis of Rotation.



Area of shell = $2\pi r h$

$r = x$
$h = y = 2x - x^2$

dx Integral.

$$V = \int_0^2 2\pi x(2x - x^2)\,dx = 2\pi \int_0^2 (2x^2 - x^3)\,dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]\Bigg|_0^2 = 2\pi \left[\frac{16}{3} - \frac{16}{4}\right] = 2\pi\left(\frac{16}{3} - 4\right)$$

$$= 2\pi\left(\frac{16}{3} - \frac{12}{3}\right) = 2\pi\left(\frac{4}{3}\right) = \frac{8\pi}{3}$$

washers |

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around x-axis.

$y = x^2 \longrightarrow x = \sqrt{y}$

$y^2 = 8x \qquad x = \frac{y^2}{8}$

$y = x^2$

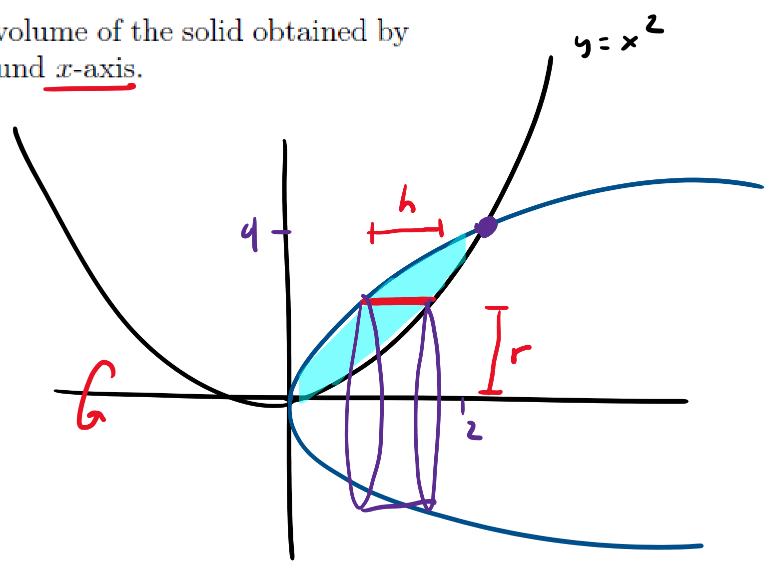
$y^2 = 8x$

$x^4 = 8x$

$x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x = 0 \quad x = 2$



Lets use shells —

dy Integral

Top − Bottom

$r = y = y - 0$

Right − Left

$h = \sqrt{y} - \frac{y^2}{8}$

$$V = \int_0^4 2\pi y \left(\sqrt{y} - \frac{y^2}{8}\right) dy$$

Shells $\int_{a}^{b} 2\pi r h \, dy$

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around y-axis.

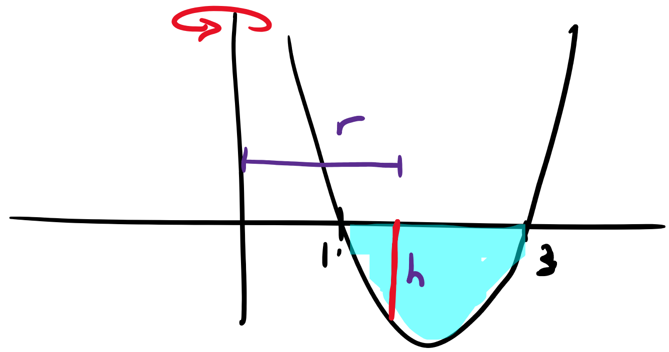
$y = x^2 - 4x + 3$ $= (x-3)(x-1)$
x−axis

use shells. dx Int

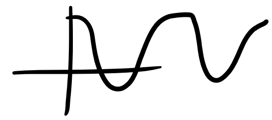
$r = x$

$h = 0 - (x^2 - 4x + 3) = -x^2 + 4x - 3$

Top Bottom



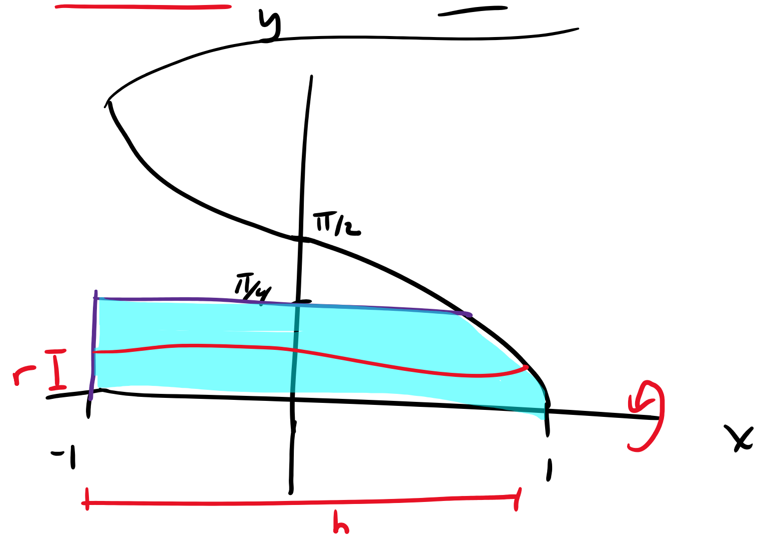
$$V = \int_1^3 2\pi x(-x^2 + 4x - 3)\,dx = \cdots = \frac{16\pi}{3}$$



Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around x-axis on the interval $y = 0$ to $y = \frac{\pi}{4}$

$x = \cos(y)$
$x = -1$

Shells dy Integral



$r = y$

$h = \underline{\text{Right} - \text{Left}}$

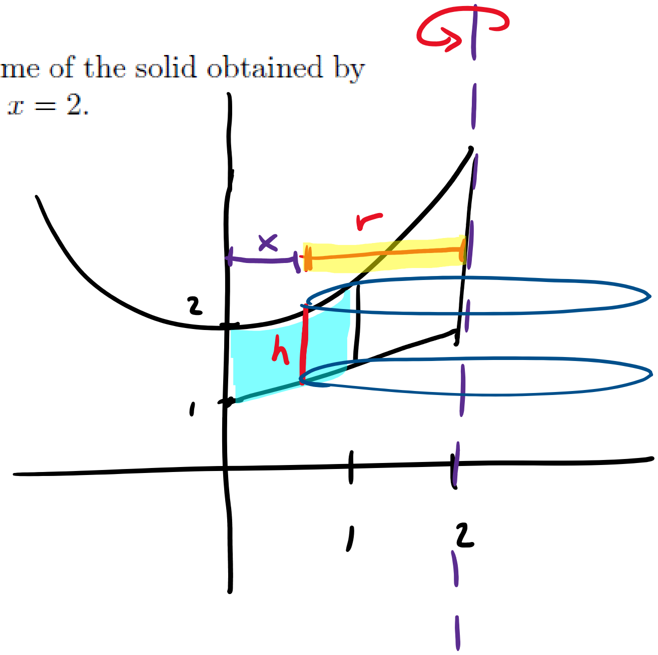
$\cos(y) - (-1)$

$= \cos(y) + 1$

$$V = \int_0^{\pi/4} 2\pi\, y\,(\cos(y) + 1)\, dy$$

$$V = 2\pi \int_0^{\pi/4} y\cos(y) + y \; dy$$

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around $x = 2$.



$y = x^2 + 2$
$2y - x = 2$ $\quad 2y = x + 2$, $\quad y = \frac{1}{2}x + 1$
$x = 0$
$x = 1$

shells. dx Integral.

$r = 2 - x$

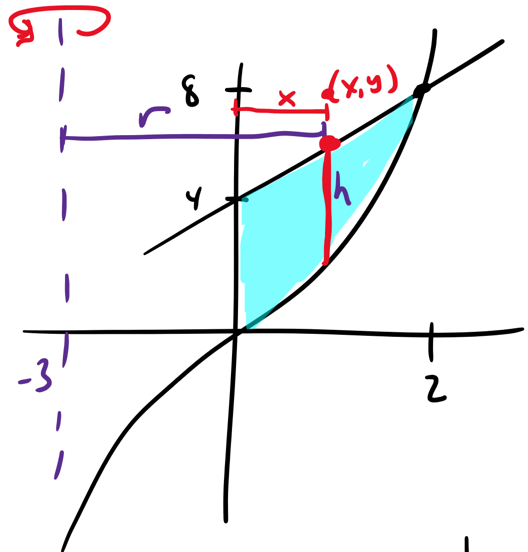
$h = x^2 + 2 - (\frac{1}{2}x + 1)$
$= x^2 - \frac{1}{2}x + 1$

$$V = \int 2\pi r h \, dx = \int_0^1 2\pi (2-x)(x^2 - \tfrac{1}{2}x + 1) \, dx$$

$x^3 = 2x+4$

Example: Set up the integral(s) that would give the volume of the solid obtained by rotating the region bounded by the given curves around $x = -3$.

$y = x^3$
$y = 2x + 4$
$x = 0$



Shell. dx Integral.

$r = x - (-3) = x + 3$

$h = 2x + 4 - x^3$

$$V = \int_0^2 2\pi (x+3)(2x+4-x^3)\,dx$$