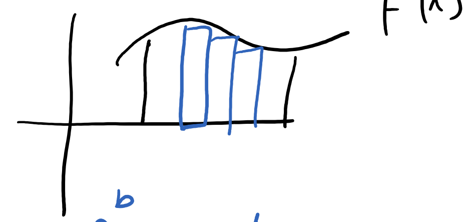
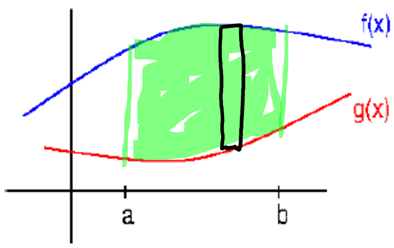
Section 6.1: Area between Curves

Consider the continuous functions $f(x)$ and $g(x)$ with the property on the interval $[a, b]$ that both are above the x-axis and $f(x) \geq g(x)$. Write down the computation that will give the area bounded between these functions on this interval.



$$\text{Area} = \int_a^b f(x)\, dx$$

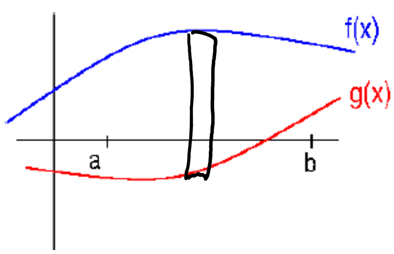
height $= f(x) - g(x)$



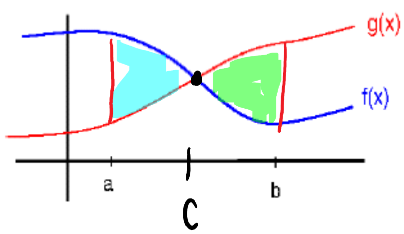
$$\int_a^b f(x) - g(x)\, dx$$

Top - bottom

For the next graphs, set-up the integral(s) that will give the area that is bounded between $f(x)$ and $g(x)$ on the interval $[a, b]$.



$$\text{Area} = \int_a^b f(x) - g(x)\, dx$$



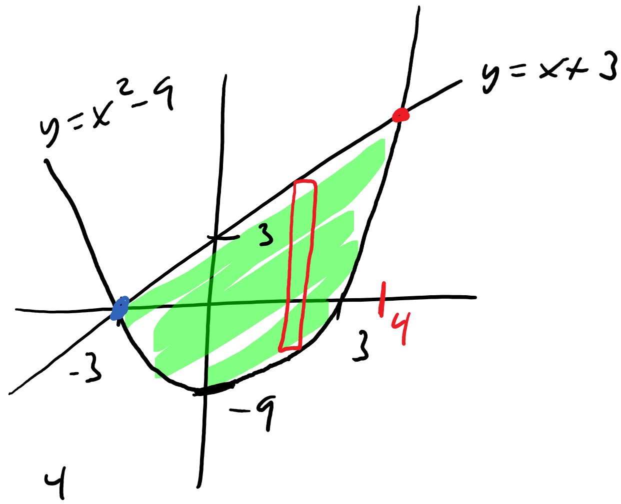
$$\text{Area} = \int_a^c f(x) - g(x)\, dx + \int_c^b g(x) - f(x)\, dx$$

$$\boxed{\text{Area} = \int_a^b |f(x) - g(x)|\, dx}$$

not counted correct

Example: Find the area that is bounded by these curves.

$y = x + 3$
$y = x^2 - 9$



$x^2 - 9 = x + 3$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \quad x = -3$

$$\text{Area} = \int_{-3}^{4} x+3 - (x^2-9) \, dx$$

$$= \int_{-3}^{4} x+3 - x^2 + 9 \, dx = \int_{-3}^{4} x - x^2 + 12 \, dx$$

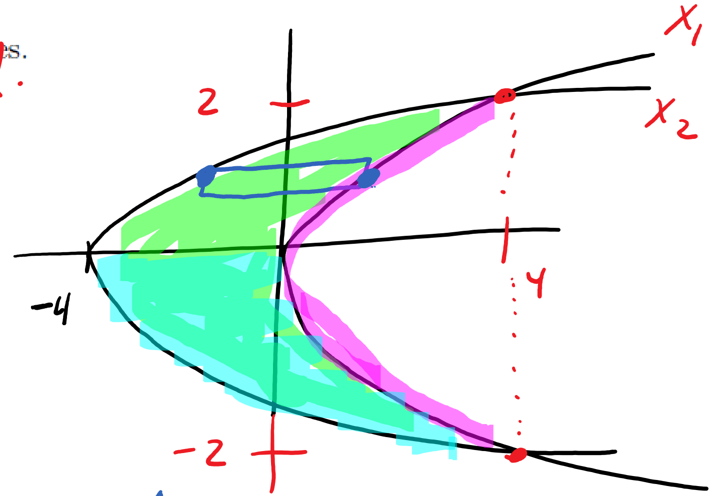
$$= \left(\frac{x^2}{2} - \frac{x^3}{3} + 12x \right) \Bigg|_{-3}^{4} = \frac{16}{2} - \frac{64}{3} + 48 - \left(\frac{9}{2} + \frac{27}{3} - 36 \right)$$

$$= \ldots\ldots = \frac{343}{6}$$

Example: Find the area that is bounded by these curves.

dy Integral.

$x_1 = y^2$
$x_2 = 2y^2 - 4$



$y^2 = 2y^2 - 4$

$4 = y^2$

$y = \pm 2$

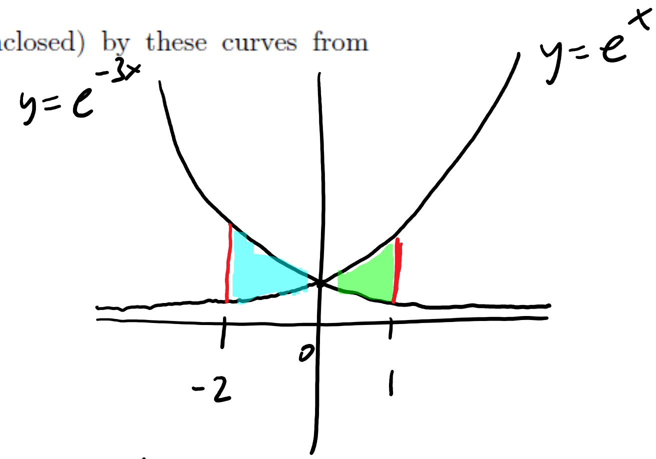
"height" = Right − Left

$$\text{Area} = \int_{-2}^{2} y^2 - (2y^2 - 4)\, dy = \int_{-2}^{2} 4 - y^2\, dy$$

$$= 2\int_{0}^{2} 4 - y^2\, dy$$

Example: Find the area that is bounded(enclosed) by these curves from $x=-2$ to $x=1$.

$y = e^{-3x}$
$y = e^{x}$



$$\text{Area} = \int_{-2}^{0} e^{-3x} - e^{x}\,dx + \int_{0}^{1} e^{x} - e^{-3x}\,dx = \dots = 134.6798$$

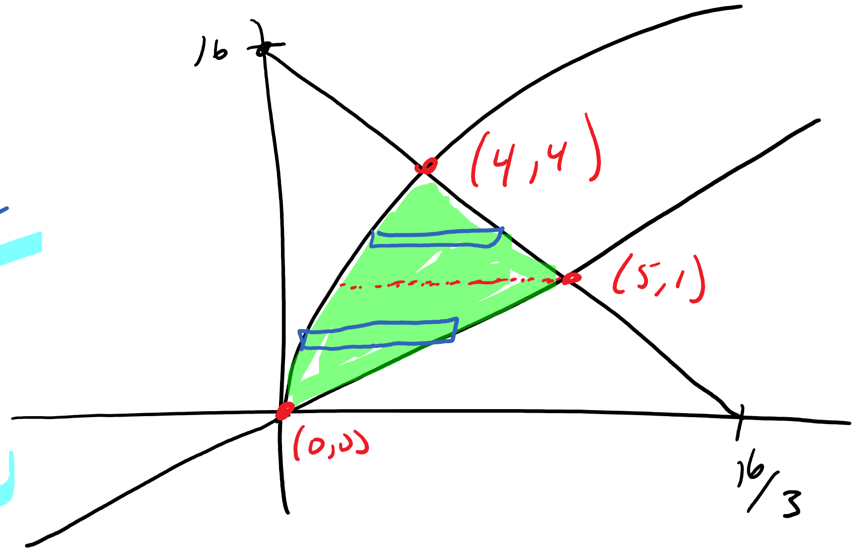
Example: Set up the integral(s), with respect to the variable y, that gives the area that is bounded(enclosed) by these curves.

$y = 2\sqrt{x}$ $\longrightarrow$ $\frac{y}{2} = \sqrt{x}$, $\frac{y^2}{4} = x$

$y = \frac{x}{5}$ $\longrightarrow$ $x = 5y$

$3x + y = 16$ $\longrightarrow$ $3x = 16 - y$, $x = \frac{16-y}{3}$

$$3x + \frac{x}{5} = 16$$
$$15x + x = 80$$
$$16x = 80$$
$$x = 5$$



$$3x + 2\sqrt{x} = 16$$
$$2\sqrt{x} = 16 - 3x$$
$$4x = 256 - 2(48)x + 9x^2$$

$$3\frac{y^2}{4} + y = 16$$
$$3y^2 + 4y = 64$$
$$3y^2 + 4y - 64 = 0$$
$$(3y + 16)(y - 4)$$
$$y = -\frac{16}{3} \qquad y = 4$$

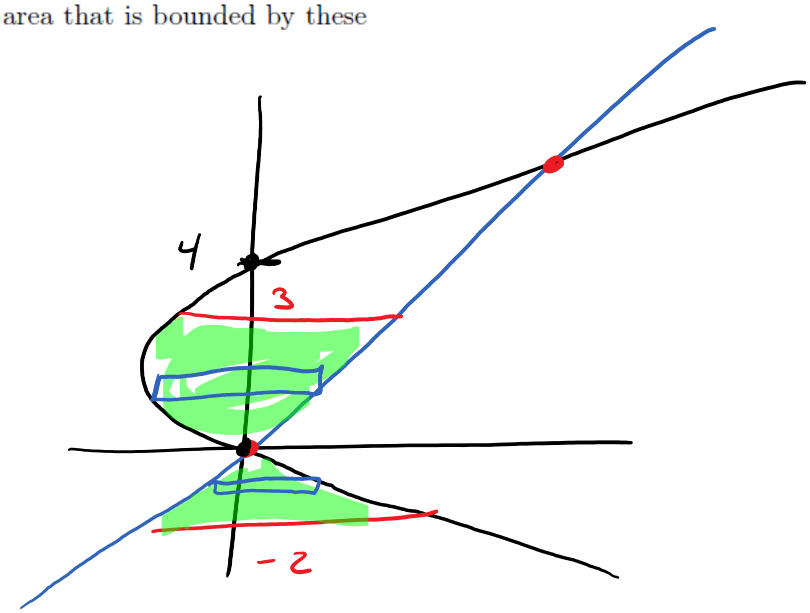
$$\text{Area} = \int_0^1 5y - \frac{y^2}{4}\, dy + \int_1^4 \frac{16-y}{3} - \frac{y^2}{4}\, dy$$

$$= \cdots = \frac{32}{3}$$

Example: Set up the integral(s) that will give area that is bounded by these curves on the interval $-2 \leq y \leq 3$.

$x = y^2 - 4y = y(y-4)$

$y = 0.5x \rightarrow y = \frac{1}{2}x \rightarrow x = 2y$



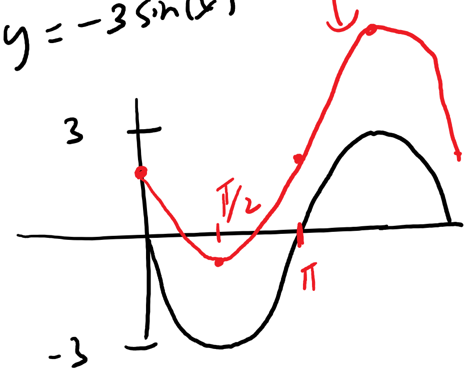
$$\text{Area} = \int_{-2}^{0} y^2 - 4y - 2y \, dy + \int_{0}^{3} 2y - (y^2 - 4y) \, dy$$

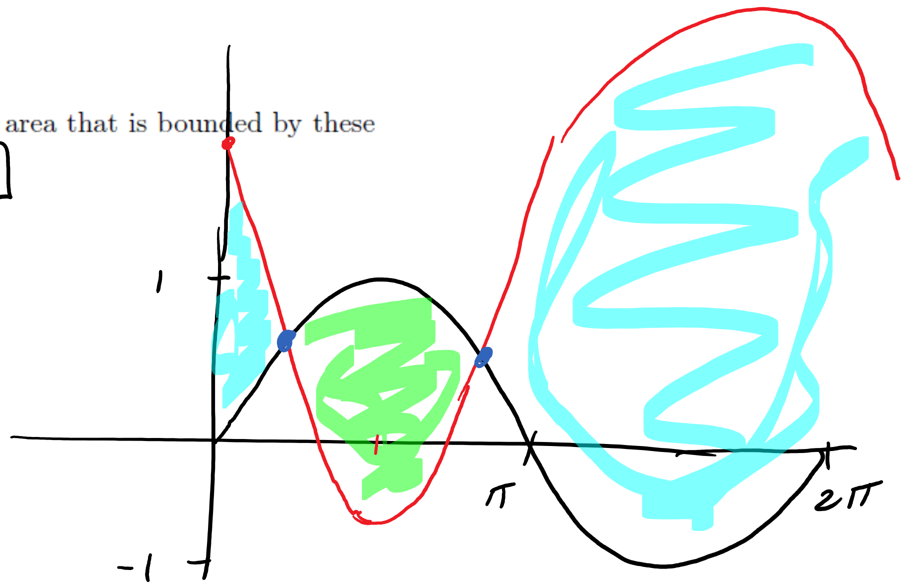
$$= \cdots = \frac{98}{3}$$

Example: Set up the integral(s) that will give area that is bounded by these curves from $x = 0$ to $x = 2\pi$. $[0, 2\pi]$

$y = \sin(x)$
$y = 2 - 3\sin(x)$

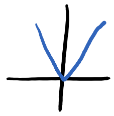
$y = -3\sin(x)$





$$\sin(x) = 2 - 3\sin(x)$$
$$4\sin(x) = 2$$
$$\sin(x) = \frac{2}{4} = \frac{1}{2}$$
$$x = \pi/6 \qquad x = 5\pi/6$$

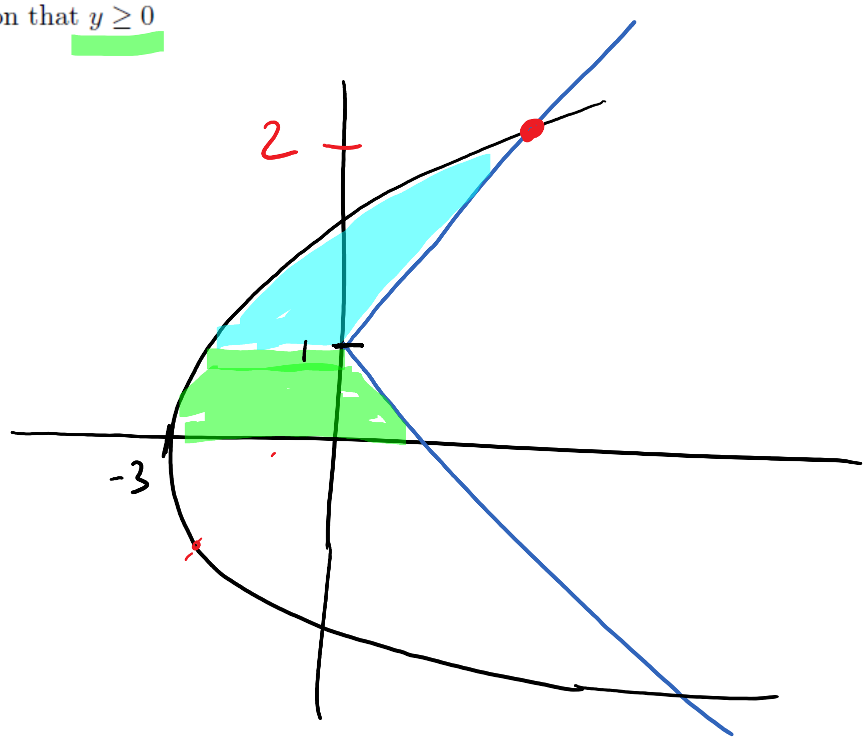
$$\text{Area} = \int_0^{\pi/6} 2 - 3\sin(x) - \sin(x)\,dx + \int_{\pi/6}^{5\pi/6} \sin(x) - [2 - 3\sin(x)]\,dx$$
$$+ \int_{5\pi/6}^{2\pi} 2 - 3\sin(x) - \sin(x)\,dx$$



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Example: Set up the integral(s) that will give area that is bounded by these curves $x = |y-1|$ and $x = y^2 - 3$ with the condition that $y \geq 0$

$x = y^2 - 3$



$x = |y-1|$

$$x = \begin{cases} y-1, & y \geq 1 \\ -(y-1), & y < 1 \end{cases}$$

$y - 1 = y^2 - 3$

$0 = y^2 - y - 2$

$0 = (y-2)(y+1)$

$y = 2 \quad y = -1$

$$\int_0^1 -(y-1) - (y^2-3)\, dy$$

$$+ \int_1^2 y-1 - (y^2-3)\, dy$$

$$= \ldots = 13/3$$