Section 7.1: Integration by Parts

Product rule: $\frac{d}{dx} f * g = f' * g + f * g'$

$$\int \frac{d}{dx} fg\,dx = \int f'g + fg' \, dx$$

$$fg = \int f'g\,dx + \int fg'\,dx$$

$$\int fg'\,dx = fg - \int f'g\,dx$$

$f = u$
$g' = dv$

$$\int u\,dv = uv - \int v\,du$$

Example: Compute the following integrals.

A) $\int xe^{2x}\,dx$

$\int xe^{2x^2}\,dx \to$ u-sub

$\int u\,dv = uv - \int v\,du$

$u = e^{2x}$ $\quad dv = x$

$du = 2e^{2x}$ $\quad V = \frac{x^2}{2}$

$\int xe^{2x}dx = \frac{x^2}{2}e^{2x} - \int \frac{x^2}{2}\cdot 2e^{2x}\,dx$

$u = x$ $\quad dv = e^{2x}$

$du = 1$ $\quad V = \frac{1}{2}e^{2x}$

$\int xe^{2x}dx = x\cdot\frac{1}{2}e^{2x} - \int 1\cdot\frac{1}{2}e^{2x}\,dx$

$= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$

B) $\int x^2 e^{2x}\,dx$

$$\int u\,dv = uv - \int v\,du$$

$u = x^2 \qquad dv = e^{2x}$

$du = 2x \qquad v = \frac{1}{2}e^{2x}$

$$\int x^2 e^{2x}\,dx = \frac{x^2}{2}e^{2x} - \int 2x \cdot \frac{1}{2}e^{2x}\,dx$$

$$= \frac{x^2}{2}e^{2x} - \int x e^{2x}\,dx$$

$u = x \qquad dv = e^{2x}$

$du = 1 \qquad V = \frac{1}{2}e^{2x}$

$$\int x^2 e^{2x}\,dx = \frac{x^2}{2}e^{2x} - \left[\frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx\right]$$

$$= \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \int \frac{1}{2}e^{2x}\,dx$$

$$\int x^2 e^{2x}\,dx = \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \frac{1}{4}e^{2x} + C$$

Page 4: tabular method

$\int u\,dv = uv - \int v\,du$

Example: Compute the following integrals. Tabular Method Used.

A) $\int xe^{2x}\,dx$

	D		I	
$u =$	x	$+$	e^{2x}	$= dv$
$du =$	1	$-$	$\frac{1}{2}e^{2x}$	$= v$
	0	$+\int$	$\frac{1}{4}e^{2x}$	

$$\int xe^{2x}\,dx = x\tfrac{1}{2}e^{2x} - 1\cdot\tfrac{1}{4}e^{2x} + \int 0\cdot\tfrac{1}{4}e^{2x}\,dx$$

$$= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$$

B) $\int x^2 e^{2x}\, dx = \frac{x^2}{2} e^{2x} - \frac{2x}{4} e^{2x} + \frac{2}{8} e^{2x} + C$

D		I
x^2	$+$	e^{2x}
$2x$	$-$	$\frac{1}{2} e^{2x}$
2	$+$	$\frac{1}{4} e^{2x}$
0	$-\int$	$\frac{1}{8} e^{2x}$

$$= \frac{x^2}{2} e^{2x} + \frac{x}{2} e^{2x} + \frac{1}{4} e^{2x} + C$$

C) $\int (x^3+6x)\sin(2x)\,dx = J$

D		I
x^3+6x		$\sin(2x)$
	$+$	
$3x^2+6$		$-\frac{1}{2}\cos(2x)$
	$-$	
$6x$		$-\frac{1}{4}\sin(2x)$
	$+$	
6		$\frac{1}{8}\cos(2x)$
	$-$	
0		$\frac{1}{16}\sin(2x)$
	$+\int$	

$$J = -\frac{(x^3+6x)}{2}\cos(2x) + \frac{3x^2+6}{4}\sin(2x) + \frac{6x}{8}\cos(2x) - \frac{6}{16}\sin(2x) + C$$

D) $\int \ln(x)\,dx = \int 1\,\ln(x)\,dx = x\ln(x) - \int \frac{1}{x}\cdot x\,dx$

$$= x\ln(x) - \int 1\,dx$$

$$= x\ln(x) - x + C$$

D		I
$\ln(x)$	$+$	1
$\frac{1}{x}$	$-\int$	x

E) $\int x^2 (\ln(x))^2 \, dx$ $= \frac{x^3}{3}(\ln(x))^2 - \frac{2}{3}\int x^2 \ln(x) \, dx$

D	I
$(\ln(x))^2$	x^2
$\frac{2\ln(x)}{x}$	$\frac{x^3}{3}$

$+$, $-\int$

D	I
$\ln(x)$	x^2
$\frac{1}{x}$	$\frac{x^3}{3}$

$+$, $-\int$

$$\int x^2(\ln(x))^2 dx = \frac{x^3}{3}(\ln(x))^2 - \frac{2}{3}\left[\frac{x^3}{3}\ln(x) - \int \frac{x^2}{3} dx\right]$$

$$= \frac{x^3}{3}(\ln(x))^2 - \frac{2x^3}{9}\ln(x) + \frac{2}{3}\cdot\frac{x^3}{9} + C$$

F) $\int x \tan^2(x)\, dx$

D	I
x	$\tan^2(x) = \sec^2(x) - 1$
1	$\tan(x) - x$
0	$-\ln\lvert\cos(x)\rvert - \frac{x^2}{2}$

$+$, $-$, $+\int$

$$\int \tan(x)\, dx = \int \frac{\sin(x)}{\cos(x)}\, dx$$

$u = \cos(x)$

$du = -\sin(x)\, dx$

$$= \int -\frac{1}{u}\, du$$

$$= -1 \ln\lvert u\rvert$$

$$= -\ln\lvert\cos(x)\rvert$$

$$\int x \tan^2(x)\, dx = x\left(\tan(x) - x\right) - \left(-\ln\lvert\cos(x)\rvert - \frac{x^2}{2}\right) + C$$

$$= x\tan(x) - x^2 + \ln\lvert\cos(x)\rvert + \frac{x^2}{2} + C$$

G) $\int 1 \arctan(x)\,dx = x\arctan(x) - \int \frac{x}{1+x^2}\,dx$

	D		I
	$\arctan(x)$	$+$	1
	$\frac{1}{1+x^2}$	$-\int$	x

$u = 1+x^2$

$= x\arctan(x) - \frac{1}{2}\ln|1+x^2| + C$

Loop

H) $\int \sin(x)\cos(3x)\,dx = J$

D		I
$\cos(3x)$	$+$	$\sin(x)$
$-3\sin(3x)$	$-$	$-\cos(x)$
$-9\cos(3x)$	$+\int$	$-\sin(x)$

$$J = -\cos(3x)\cos(x) - 3\sin(3x)\sin(x) + \int 9\cos(3x)\sin(x)\,dx$$

$$J = -\cos(3x)\cos(x) - 3\sin(3x)\sin(x) + 9\int \cos(3x)\sin(x)\,dx$$

$$J = -\cos(3x)\cos(x) - 3\sin(3x)\sin(x) + 9J$$

$$-8J = -\cos(3x)\cos(x) - 3\sin(3x)\sin(x)$$

$$J = \frac{1}{8}\cos(3x)\cos(x) + \frac{3}{8}\sin(3x)\sin(x) + C$$

I) $\int \sin(x)e^{3x}\,dx = J$

D		I
$\sin(x)$	$+$	e^{3x}
$\cos(x)$	$-$	$\frac{1}{3}e^{3x}$
$-\sin(x)$	$+\int$	$\frac{1}{9}e^{3x}$

$$J = \frac{1}{3}\sin(x)e^{3x} - \frac{1}{9}\cos(x)e^{3x} - \frac{1}{9}\overbrace{\int \sin(x)e^{3x}\,dx}^{J}$$

$$1J = [\quad] - \frac{1}{9}J$$

$$\frac{10}{9}J = [\quad]$$

$$J = \frac{9}{10}[\quad] + C$$

$$J = \frac{9}{10}\left[\frac{1}{3}\sin(x)e^{3x} - \frac{1}{9}\cos(x)e^{3x}\right] + C$$

J) $\int x^3 \sin(x^2)\, dx = \int x^2 \cdot x \sin(x^2)\, dx = \int \frac{1}{2} u \sin(u)\, du$

$u = x^2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

D	I
$\frac{1}{2}u$	$\sin(u)$
$\frac{1}{2}$	$-\cos(u)$
0	$-\sin(u)$

$+$

$-$

$+\int$

$= -\frac{1}{2} u \cos(u) + \frac{1}{2} \sin(u) + C$

$= -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \sin(x^2) + C$