152 Week In Review: Sections 5.5-6.1

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

1. Integrate the following: Note k is some non-zero constant.

$\int e^x dx = e^x$

(a) $\int e^{kx}\, dx = \int \frac{1}{K} e^u du$

$u = Kx$

$du = K dx$

$\frac{1}{K} du = dx$

$= \frac{1}{K} e^u + C$

$= \frac{1}{K} e^{Kx} + C = \int e^{Kx} dx$

(b) $\int \cos(kx)\, dx$

$= \frac{1}{K} \sin(Kx) + C$

(c) $\int \sin(kx)\, dx = -\frac{1}{K} \cos(Kx) + C$

(junk) power

2. Integrate the following.

(a) $\int 5x^2(x^3+7)^8\,dx$ $= \int 5x^2 u^8 \cdot \frac{1}{3x^2}\,du = \int \frac{5}{3} u^8\,du$

$u = x^3+7$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$\frac{5}{3}du = 5x^2 dx$

$= \frac{5}{3}\,\frac{u^9}{9} + C$

$= \frac{5}{27}(x^3+7)^9 + C$

$\frac{\text{something}}{\text{junk}}$

(b) $\int \frac{x^2+2}{x^3+6x}\,dx = \int \frac{x^2+2}{u} \frac{1}{3x^2+6}\,du$

$u = x^3+6x$

$du = (3x^2+6)\,dx$

$\frac{1}{3x^2+6}\,du = dx$

$= \int \frac{x^2+2}{u} \cdot \frac{1}{3(x^2+2)}\,du$

$= \int \frac{1}{3u}\,du = \int \frac{1}{3} \cdot \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|x^3+6x| + C$

e^{junk}

(c) $\int (6x^2+8)e^{x^3+4x}\,dx$ $= \int (6x^2+8)e^u \cdot \frac{1}{3x^2+4}\,du$

$u = x^3+4x$

$du = (3x^2+4)\,dx$

$\frac{1}{3x^2+4}\,du = dx$

$= \int \frac{2(3x^2+4)e^u}{(3x^2+4)}\,du$

$= \int 2e^u\,du$

$= 2e^u + C$

$= 2e^{x^3+4x} + C$

(d) $\int 7e^{3x}\sin(e^{3x})\,dx$ $= \int 7e^{3x}\sin(u)\cdot\frac{1}{3e^{3x}}\,du$

trig(junk)

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3e^{3x}}\,du = dx$

$= \int \frac{7}{3}\sin(u)\,du$

$= -\frac{7}{3}\cos(u) + C$

$= -\frac{7}{3}\cos(e^{3x}) + C$

(e) $\int \frac{\sec^2(x^{-3})}{x^4}\,dx = \int \frac{\sec^2(u)}{x^4} \frac{-x^4}{3}\,du$

$u = x^{-3}$

$du = -3x^{-4}\,dx$

$du = \frac{-3}{x^4}\,dx$

$-\frac{x^4}{3}\,du = dx$

$= \int -\frac{1}{3}\sec^2(u)\,du$

$= -\frac{1}{3}\tan(u) + C$

$= -\frac{1}{3}\tan(x^{-3}) + C$

(f) $\int \frac{5+4x}{x^2+1}\,dx$ $= \underbrace{\int \frac{4x}{x^2+1}\,dx}$ $+ \int \frac{5}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2x}\,du = dx$

$= \int \frac{4x}{u} \cdot \frac{1}{2x}\,du + \int \frac{5}{x^2+1}\,dx$

$= \int \frac{2}{u}\,du + 5\arctan(x) + C$

$= 2\ln|u| + 5\arctan(x) + C$

$= 2\ln|x^2+1| + 5\arctan(x) + C$

(g) $\int x\sqrt{x+5}\,dx = \int x\sqrt{u}\,du = \int (u-5)\sqrt{u}\,du$

$\boxed{u = x+5} \rightarrow x = u-5$

$du = dx$

$= \int (u-5)\,u^{1/2}\,du$

$= \int u^{3/2} - 5u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} - 5\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x+5)^{5/2} - \frac{10}{3}(x+5)^{3/2} + C$

(h) $\int_0^{\pi/4} \frac{1}{\cos^2(x)\sqrt{1+\tan(x)}}\,dx$ $= \int_1^2 \frac{1}{\cos^2(x)\sqrt{u}} \cos^2(x)\,du$

$u = 1 + \tan(x)$

$du = \sec^2(x)\,dx$

$\frac{1}{\sec^2(x)}\,du = dx$

$\cos^2(x)\,du = dx$

$x = 0$

$u = 1 + \tan(0)$

$u = 1 + 0$

$u = 1$

$x = \pi/4$

$u = 1 + \tan\left(\frac{\pi}{4}\right)$

$u = 1 + 1$

$u = 2$

$= \int_1^2 \frac{1}{\sqrt{u}}\,du$

$= \int_1^2 u^{-1/2}\,du$

$= \frac{2}{1} u^{1/2} \Big|_1^2$

$= 2\sqrt{2} - 2\sqrt{1}$

$= \underline{\underline{2\sqrt{2} - 2}}$

(i) $\int_1^2 x^3(1-x^2)^5\,dx$ $= \int_0^{-3} x^3 u^5 \frac{-1}{2x}\,du = -\frac{1}{2}\int_0^{-3} x^2 u^5\,du$

$\boxed{u = 1-x^2}$ $\rightarrow$ $x^2 = 1-u$

$du = -2x\,dx$

$-\frac{1}{2x}\,du = dx$

$\underline{\underline{x=1}}$

$u = 1-1^2 = 0$

$\underline{x=2}$

$u = 1-2^2$

$u = 1-4$

$u = -3$

$= -\frac{1}{2}\int_0^{-3} (1-u)\,u^5\,du$

$= -\frac{1}{2}\int_0^{-3} u^5 - u^6\,du$

$= -\frac{1}{2}\left[\frac{u^6}{6} - \frac{u^7}{7}\right]_0^{-3}$

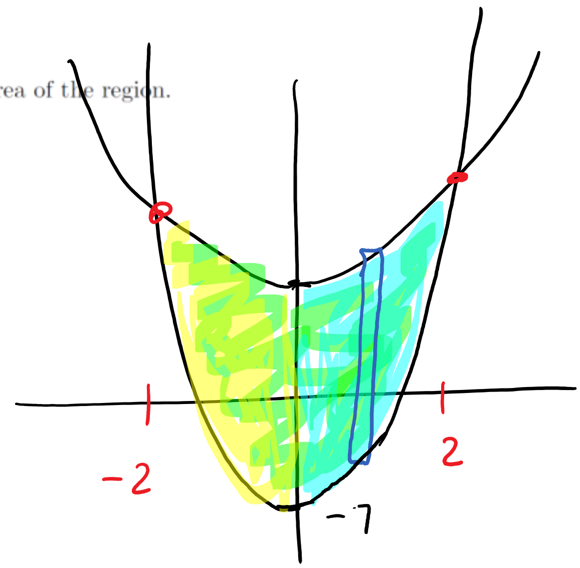
$= -\frac{1}{2}\left[\frac{(-3)^6}{6} - \frac{(-3)^7}{7}\right] - \frac{1}{2}\left[0-0\right]$

$= -\frac{1}{2}\left[\frac{(-3)^6}{6} - \frac{(-3)^7}{7}\right]$

3. Sketch the region enclosed by these curves, and then find the area of the region.

$y = 2x^2 + 5$

$y = 5x^2 - 7$



dx Integral

Intersect

$2x^2 + 5 = 5x^2 - 7$

$12 = 3x^2$

$4 = x^2$

$x = \pm 2$

dx Integral

$$\text{Area} = \int_a^b \text{top} - \text{bottom}\ dx$$

$$A = \int_{-2}^{2} (2x^2+5) - (5x^2-7)\ dx$$

$$= \int_{-2}^{2} 2x^2 + 5 - 5x^2 + 7\ dx$$

$$= \int_{-2}^{2} 12 - 3x^2\ dx = 2\int_0^2 12 - 3x^2\ dx$$

$$= 2\left[12x - x^3\right]_0^2 = 2\left[(24-8) - (0)\right]$$

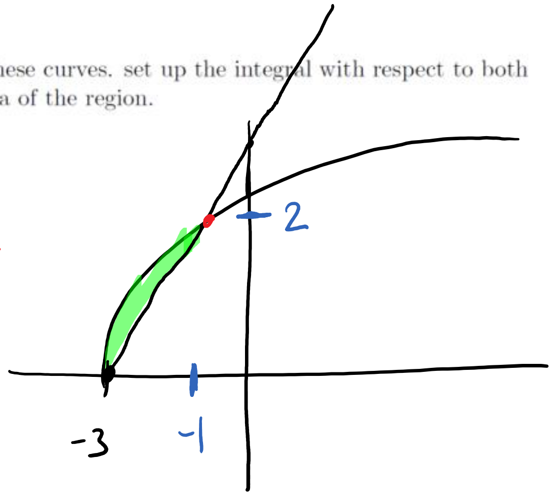
$$= 2[16]$$

$$= 32$$

4. Sketch the region enclosed by these curves. set up the integral with respect to both x and y that would give the area of the region.

$y = \sqrt{2x+6}$ → $y^2 = 2x+6$, $y^2 - 6 = 2x$, $x = \frac{y^2-6}{2}$

$y = x + 3$ → $x = y - 3$



Intersection

$\sqrt{2x+6} = x+3$

$2x+6 = (x+3)^2$

$2x+6 = x^2+6x+9$

$0 = x^2+4x+3$

$0 = (x+3)(x+1)$

$x = -3 \quad x = -1$

dx Integral

Top − Bottom

$$\int_{-3}^{-1} \sqrt{2x+6} - (x+3)\, dx$$

dy Integral

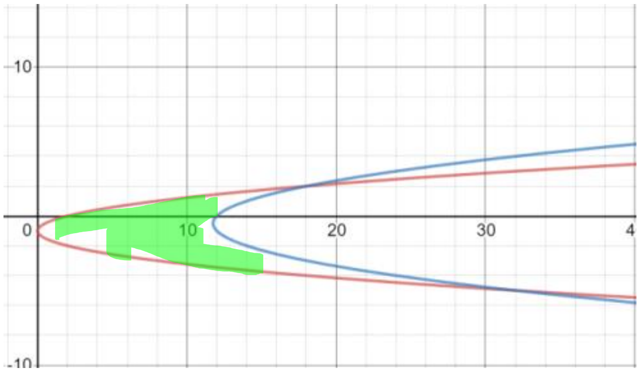
Right − Left

$$\int_0^2 y-3 - \left(\frac{y^2-6}{2}\right) dy$$

5. Setup the integral(s) that would find the area bounded by these curves.

$x = 2y^2 + 4y + 2$

$x = y^2 + y + 12$



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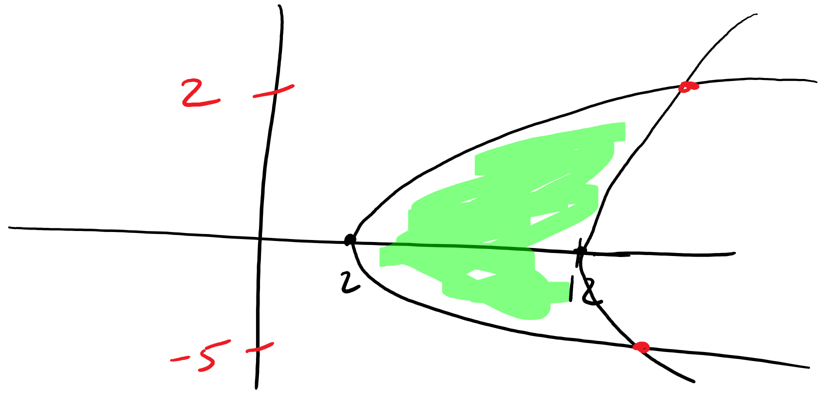


$2y^2 + 4y + 2 = y^2 + y + 12$

$y^2 + 3y - 10 = 0$

$(y+5)(y-2) = 0$

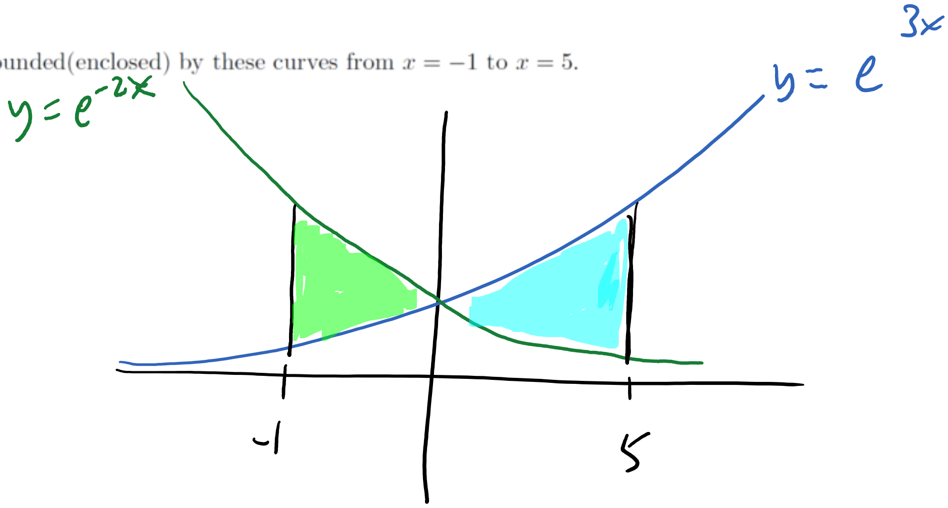
$y = -5 \quad y = 2$

$$\boxed{\int_{-5}^{2} y^2 + y + 12 - (2y^2 + 4y + 2)\, dy}$$

6. Find the area that is bounded(enclosed) by these curves from $x = -1$ to $x = 5$.

$y = e^{3x}$

$y = e^{-2x}$



$$\int_{-1}^{0} e^{-2x} - e^{3x}\,dx + \int_{0}^{5} e^{3x} - e^{-2x}\,dx$$

$$= \left(\frac{-1}{2}e^{-2x} - \frac{1}{3}e^{3x}\right)\Big|_{-1}^{0} + \left(\frac{1}{3}e^{3x} - \frac{-1}{2}e^{-2x}\right)\Big|_{0}^{5}$$

$$= \left(\frac{-1}{2} - \frac{1}{3}\right) - \left(\frac{-1}{2}e^{2} - \frac{1}{3}e^{-3}\right) + \left(\frac{1}{3}e^{15} + \frac{1}{2}e^{-10}\right) - \left(\frac{1}{3} + \frac{1}{2}\right)$$

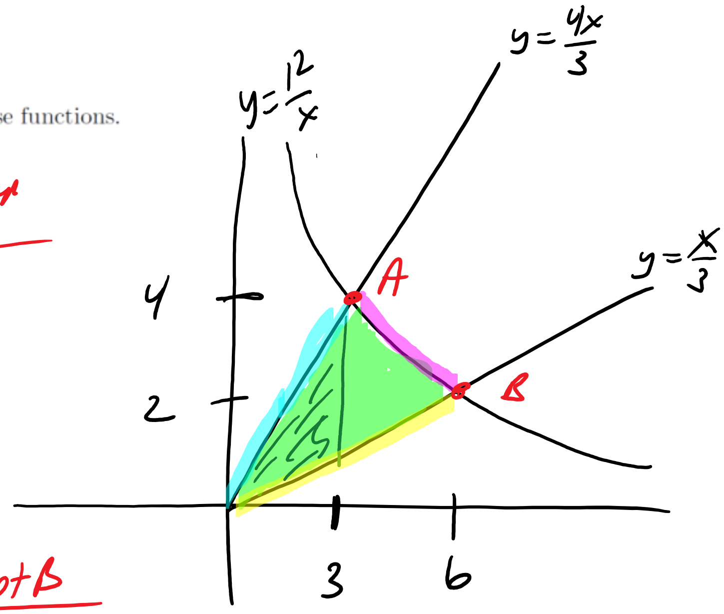
7. Find the area of the region in the first quadrant that is bounded by these functions.

$xy = 12 \longrightarrow y = \frac{12}{x}$

$3y = x \longrightarrow y = \frac{x}{3}$

$3y = 4x \longrightarrow y = \frac{4x}{3}$

dx Integral



Intersection pt A

$y = \frac{4x}{3} \qquad y = \frac{12}{x}$

$\frac{4x}{3} = \frac{12}{x}$

$4x^2 = 36$

$x^2 = 9$

$x = \pm 3$

Intersection pt B

$y = \frac{x}{3} \qquad y = \frac{12}{x}$

$\frac{x}{3} = \frac{12}{x}$

$x^2 = 36$

$x = \pm 6$

dy Integral

$$\int_0^3 \frac{4x}{3} - \frac{x}{3}\, dx + \int_3^6 \frac{12}{x} - \frac{x}{3}\, dx$$

$$= \int_0^3 x\, dx + \int_3^6 \frac{12}{x} - \frac{x}{3}\, dx$$

$$= \frac{x^2}{2}\Big|_0^3 + \left(12\ln|x| - \frac{x^2}{6}\right)\Big|_3^6$$

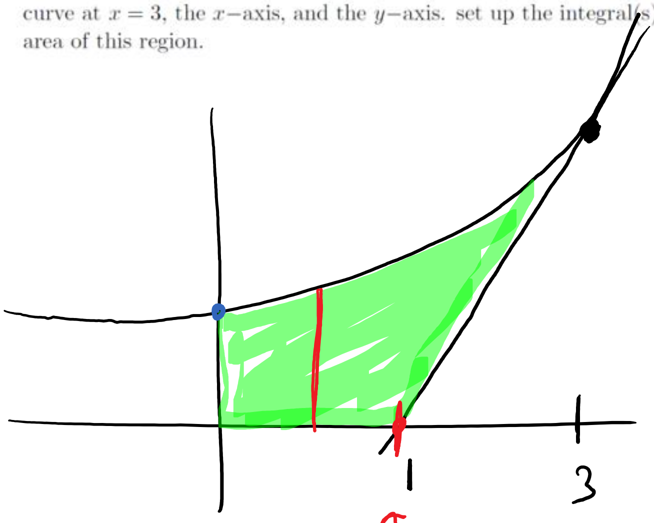
$$= \frac{9}{2} + 12\ln(6) - \frac{36}{6} - \left(12\ln(3) - \frac{9}{6}\right)$$

$$= \frac{9}{2} + 12\ln(6) - \frac{36}{6} - \left(12\ln(3) \quad 6\right)$$

$$= \frac{9}{2} + 12\ln(6) - 6 - 12\ln(3) + \frac{3}{2}$$

$$= 12\ln(6) - 12\ln(3) = \ldots\ldots = 12\ln(2)$$

8. Sketch the region that is bounded by the curve $y = e^{x/2}$, the tangent line to this curve at $x = 3$, the x-axis, and the y-axis. set up the integral(s) that will find the area of this region.



$$y = e^{x/2}$$

$$y' = \frac{1}{2}e^{x/2}$$

$$m_{tan} = \frac{1}{2}e^{3/2}$$

$$y - e^{3/2} = \frac{1}{2}e^{3/2}(x-3)$$

$$y - e^{3/2} = \frac{1}{2}xe^{3/2} - \frac{3}{2}e^{3/2}$$

$$\underbrace{y = \frac{1}{2}xe^{3/2} - \frac{1}{2}e^{3/2}}_{\text{tangent line.}}$$

$$0 = \frac{1}{2}xe^{3/2} - \frac{1}{2}e^{3/2}$$

$$\frac{1}{2}e^{3/2} = \frac{1}{2}xe^{3/2}$$

$$1 = x$$

Area

$$\int_0^1 e^{x/2}\,dx + \int_1^3 e^{\frac{x}{2}} - \left(\frac{1}{2}xe^{3/2} - \frac{1}{2}e^{3/2}\right)dx$$