152 Week In Review: Sections 6.2

Solutions and questions can be found at the link:
https://www.math.tamu.edu/~kahlig/152WIR.html

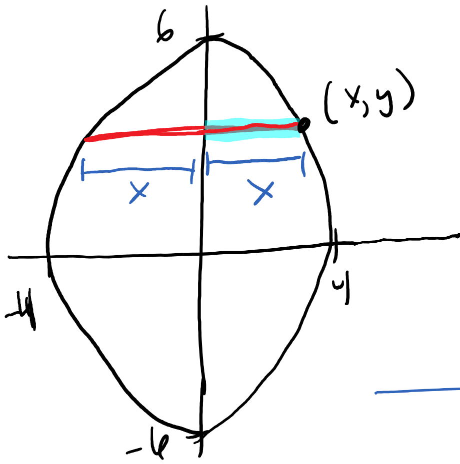
Volume of a solid: $V = \int_a^b A(x)\,dx$ or $V = \int_a^b A(y)\,dy$

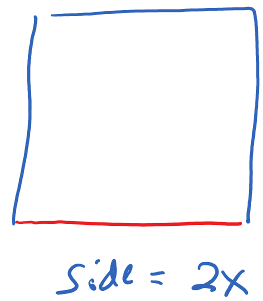
$A(x)$ represents the area of the slice and the slice is perpendicular to the $x-$axis.

$A(y)$ represents the area of the slice and the slice is perpendicular to the $y-$axis.

1. The base of a solid is the region enclosed by the ellipse $\frac{x^2}{16} + \frac{y^2}{36} = 1$. Cross-sections perpendicular to the y−axis are squares. Find the volume of the solid.

dy Integral





$A = (\text{side})^2$

$A = (2x)^2 = 4x^2$

$A = 4\left(16 - \frac{16y^2}{36}\right)$

$A = 64 - \frac{64}{36}y^2$

$A = 64 - \frac{16}{9}y^2$

$\frac{x^2}{16} = 1 - \frac{y^2}{36}$

$x^2 = 16 - 16y^2/36$

$$V = \int_{-6}^{6} 64 - \frac{16}{9}y^2\,dy = 2\int_0^6 64 - \frac{16}{9}y^2\,dy$$

$$= 2\left[64y - \frac{16}{9}\frac{y^3}{3}\right]_0^6$$

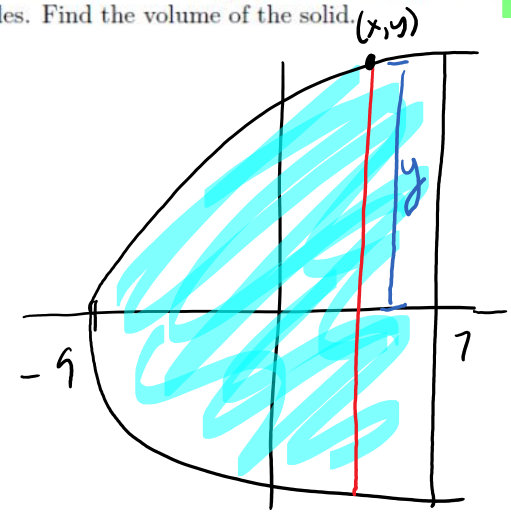
$$= 2\left[64(6) - \frac{16}{27}(6)^3 - (0)\right]$$

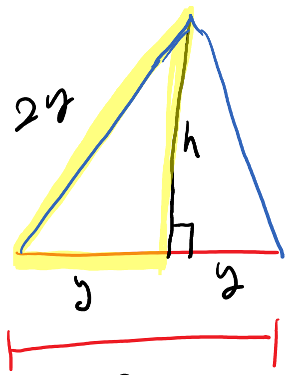
$$= 2(256) = 512$$

dx Integral

2. The base of a solid is the region enclosed by $x = y^2 - 9$ and the $x = 7$. Cross-sections perpendicular to the $x-$axis are equilateral triangles. Find the volume of the solid.

$y^2 = x + 9$





$b = 2y$

$y^2 + h^2 = (2y)^2$

$y^2 + h^2 = 4y^2$

$h^2 = 3y^2$

$h = y\sqrt{3}$

$A = \frac{1}{2} b h = \frac{1}{2}(2y)\, y\sqrt{3}$

$A = y^2\sqrt{3} = (x+9)\sqrt{3}$

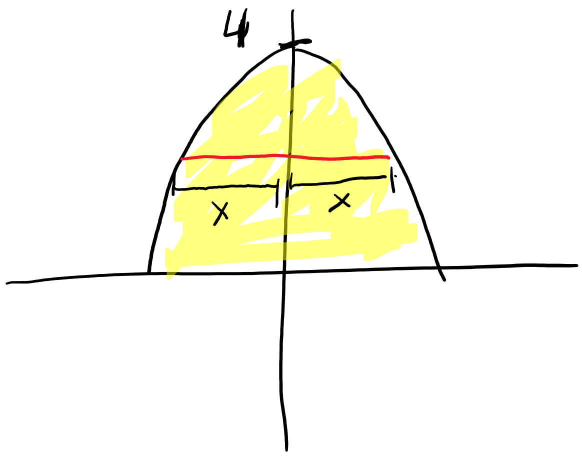
$$V = \int_{-9}^{7} (x+9)\sqrt{3}\, dx = \cdots = 128\sqrt{3}$$

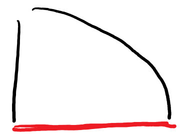
3. The base of a solid is the region enclosed by $y = 4 - x^2$ and the x-axis. Cross-sections perpendicular to the y-axis are quarter circles. Find the volume of the solid.

$y + x^2 = 4$

$x^2 = 4 - y$

dy Integral.





$r = 2x$

$$A = \frac{1}{4}\pi r^2$$

$$A = \frac{1}{4}\pi (2x)^2$$

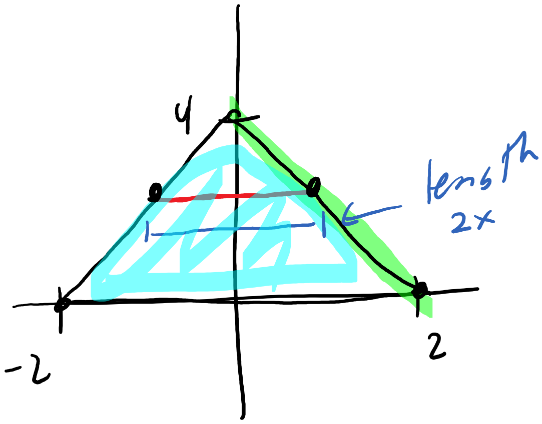
$$A = \frac{1}{4}\pi 4x^2 = \pi x^2$$

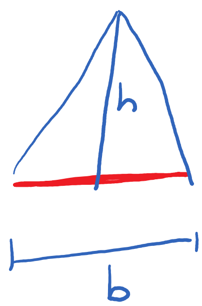
$$A = \pi(4-y)$$

$$V = \int_0^4 \pi(4-y)\,dy = \dots = 8\pi$$

dy Integral

4. The base of a solid is a triangular region with vertices (0, 4), (2, 0), and (−2, 0). Cross-sections perpendicular to the y−axis are isosceles triangles with the height equal to half the base. Find the volume of the solid.





$b = 2x$

$h = \frac{1}{2} b$

$h = \frac{1}{2}(2x) = x$

$A = \frac{1}{2} b h$

$A = \frac{1}{2}(2x)x = x^2$

$A = \left(2 - \frac{y}{2}\right)^2$

find the eq. of the line thru the points (0,4) (2,0)

$m = \frac{4-0}{0-2} = \frac{4}{-2} = -2$

$y - 4 = -2(x - 0)$

$y = -2x + 4$

$2x = 4 - y$

$x = 2 - \frac{1}{2} y$

$x_1 = 2 - \frac{1}{2} y$

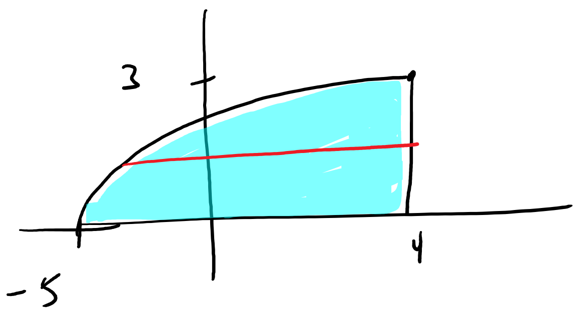
$V = \int_0^4 \left(2 - \frac{y}{2}\right)^2 dy$

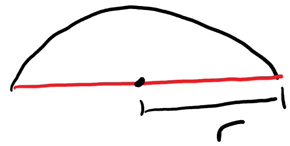
$= \dots = \frac{16}{3}$

$y^2 = 5 + x$
$x = y^2 - 5$

This is a challenging Question.

5. The base of a solid is the region enclosed by $y = \sqrt{x+5}$, $x = 4$ and $y = 0$. Cross-sections perpendicular to the y-axis are semicircles. Setup the integral to find the volume of the solid.





$A = \frac{1}{2}\pi r^2$

length of the Red line is

$4 - x$ from Right - left

$= 4 - (y^2 - 5) = 4 - y^2 + 5$

$= 9 - y^2$

dy Integral

$r = \frac{1}{2}(\text{length of the slice}) = \frac{1}{2}(9 - y^2)$

Thus $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left[\frac{1}{2}(9-y^2)\right]^2 = \frac{1}{2}\pi \cdot \frac{1}{4}(9-y^2)^2$

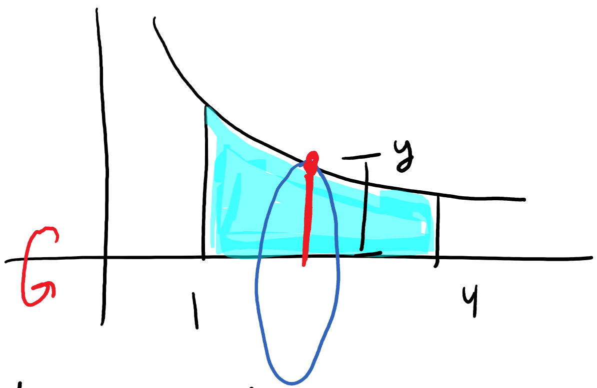
$$V = \int_0^3 \frac{\pi}{8}(9-y^2)^2\, dy = \dots = \frac{81\pi}{5}$$

6. Find the volume of the solid obtained by rotating the region bounded the curves about the x-axis.

$y = \frac{2}{x}$ $\quad x$-axis $\quad x = 1 \quad x = 4$

If the slice is perpendicular to the axis of Rotation then disk/washer method.

If the slice is parallel to the axis of rotation then shell method



$A = \pi r^2$

dx Integral

$A = \pi y^2 = \pi\left(\frac{2}{x}\right)^2 = \frac{4\pi}{x^2}$

$r = y$

$$V = \int_1^4 \frac{4\pi}{x^2}\,dx = 4\pi\int_1^4 x^{-2}\,dx$$

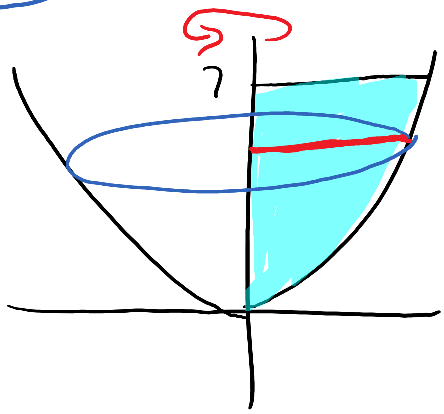
$$= 4\pi\left(-x^{-1}\right)\Big|_1^4 = \frac{-4\pi}{x}\Big|_1^4$$

$$= \frac{-4\pi}{4} - \frac{-4\pi}{1} = -\pi + 4\pi = 3\pi$$

7. Find the volume of the solid obtained by rotating the region bounded the curves about the y–axis.

$x^2 = 4y$ $\quad$ y–axis $\quad$ $y = 7$

dy Integral



$A = \pi r^2$

$r = x$ $\qquad A = \pi x^2$

$A = \pi(4y)$

$A = 4\pi y$

$$V = \int_0^7 4\pi y \, dy = \cdots = 98\pi$$

$R = r_o$ = outer radius

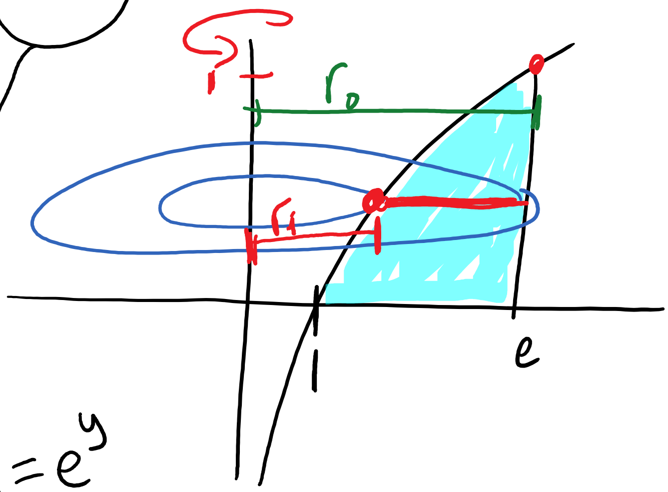
$r = r_i$ = inner radius

dy Integral

8. Find the volume of the solid obtained by rotating the region bounded the curves about the y–axis.

$y = \ln(x)$ $\quad x$–axis $\quad x = e$

$x = e^y$



$A = \pi R^2 - \pi r^2$

$\pi r_o^2 - \pi r_i^2$

$r_o = e$

$r_i = x = e^y$

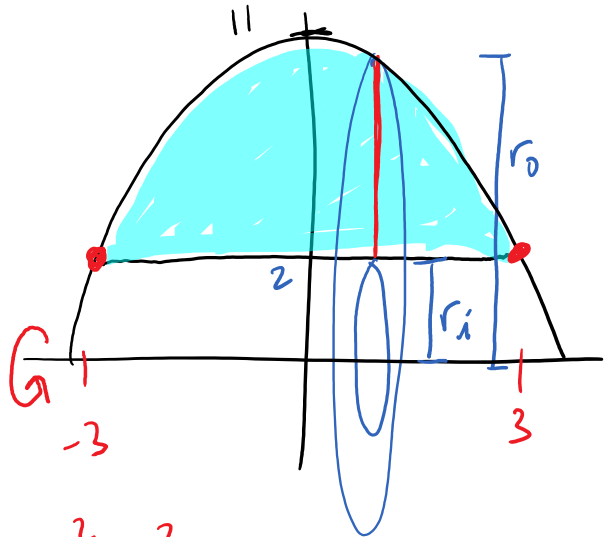
$$V = \int_0^1 \pi \left[(e)^2 - (e^y)^2 \right] dy$$

$$= \pi \int_0^1 e^2 - e^{2y} \; dy$$

$$= \cdots = \pi \left[\tfrac{1}{2} e^2 + \tfrac{1}{2} \right]$$

9. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the x−axis.

$y = 11 - x^2$ $\quad$ $y = 2$



$11 - x^2 = 2$

$11 = x^2 + 2$

$9 = x^2 \quad x = \pm 3$

dx Integral

$r_o = 11 - x^2$

$r_i = 2$

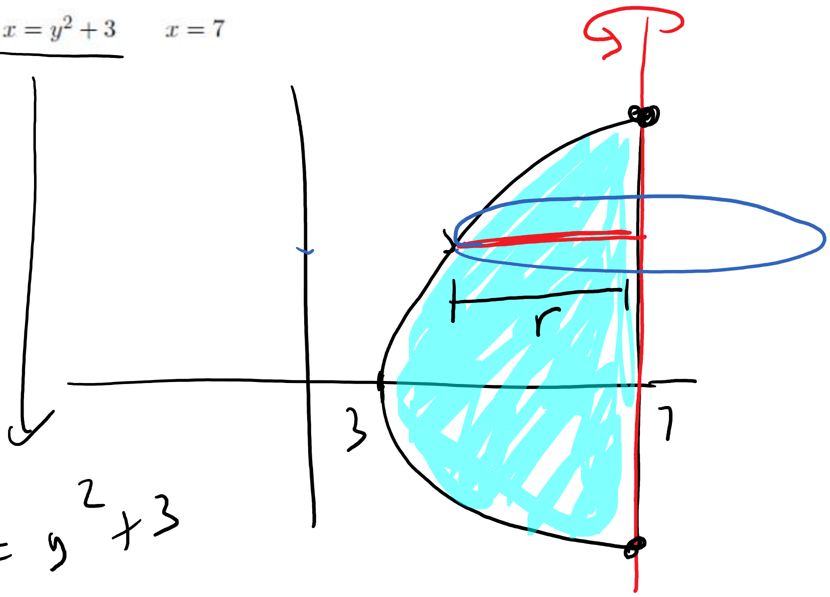
$$V = \int_{-3}^{3} \pi \left[(11-x^2)^2 - (2)^2 \right] dx$$

$$V = 2 \int_{0}^{3} \pi \left[(11-x^2)^2 - 4 \right] dx$$

dy Integral

10. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the line $x = 7$.

$x = y^2 + 3 \qquad x = 7$



Right - Left

$r = 7 - (y^2 + 3)$

$= 7 - y^2 - 3$

$= 4 - y^2$

$7 = y^2 + 3$

$4 = y^2$

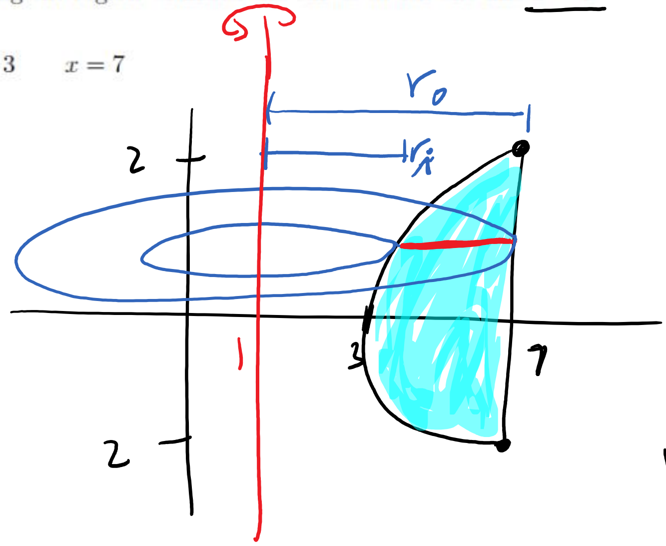
$y = \pm 2$

$$V = \int_{-2}^{2} \pi (4 - y^2)^2 \, dy$$

$$V = 2\int_{0}^{2} \pi (4 - y^2)^2 \, dy$$

11. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the line $x = 1$.

$x = y^2 + 3 \qquad x = 7$



dy Integral

right − left

$r_o = 7 - 1 = 6$

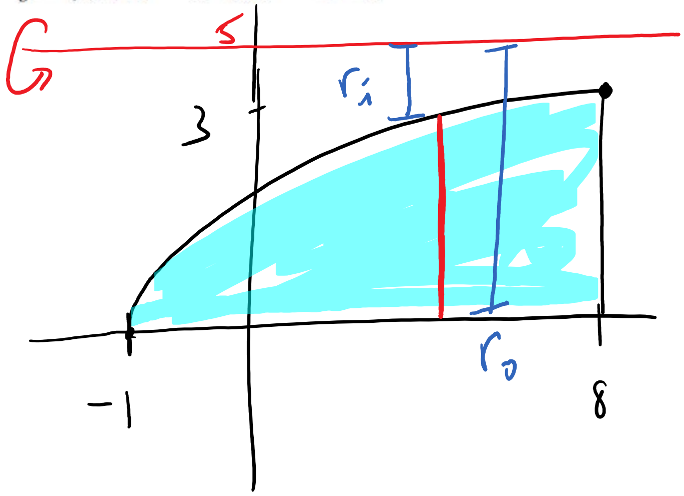
$r_i = y^2 + 3 - 1 = y^2 + 2$

$$V = \int_{-2}^{2} \pi \left[6^2 - (y^2+2)^2 \right] dy$$

$$= 2 \int_0^2 \pi \left[36 - (y^2+2)^2 \right] dy$$

12. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the line $y = 5$.

$y = \sqrt{x+1}$ $\quad$ x−axis $\quad$ $x = 8$



dx Integral

top − Bottom

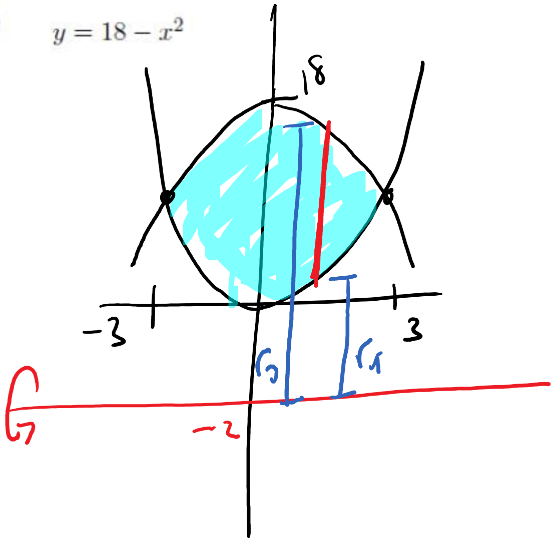
$r_o = 5 - 0 = 5$

$r_i = 5 - \sqrt{x+1}$

$$V = \int_{-1}^{8} \pi \left[5^2 - \left(5 - \sqrt{x+1}\right)^2 \right] dx$$

13. Set up the integral(s) that would find the volume of the solid obtained by rotating the region bounded the curves about the line $y = -2$.

$y = x^2 \qquad y = 18 - x^2$



dx Integral

$x^2 = 18 - x^2$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

$r_0 = 18 - x^2 - (-2)$

$= 18 - x^2 + 2$

$= 20 - x^2$

$r_1 = x^2 - (-2) = x^2 + 2$

$$V = \int_{-3}^{3} \pi \left[(20 - x^2)^2 - (x^2 + 2)^2 \right] dx$$

$$= 2 \int_{0}^{3} \pi \left[(20 - x^2)^2 - (x^2 + 2)^2 \right] dx$$