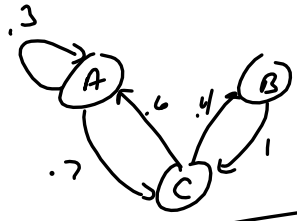
Section M.2: Regular Markov Processes

Definition: A transition matrix (stochastic matrix) is said to be **regular** if some power of T has all positive entries. This means that the Markov chain represented by T is called a regular Markov chain.

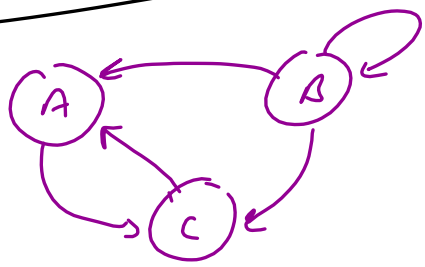
Example: Are the transition matrices regular?

A) $T = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$ Regular.



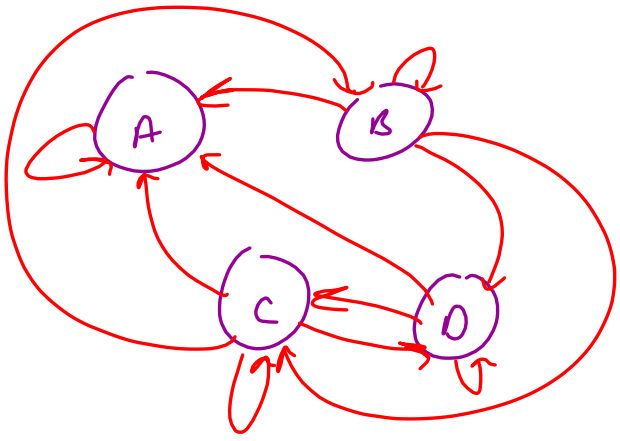
B) $T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.6 \\ 0 & 0 & 0.4 \\ 0.7 & 1 & 0 \end{bmatrix} \end{array}$ yes

C) $T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0.2 & 1 \\ 0 & 0.3 & 0 \\ 1 & 0.5 & 0 \end{bmatrix} \end{array}$ NO



D) $T = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0.2 & 0.1 & 0.1 \\ 0 & 0.4 & 0.1 & 0 \\ 0 & 0.2 & 0.2 & 0.5 \\ 0 & 0.2 & 0.5 & 0.4 \end{bmatrix} \end{array}$

NO.



Definition: A Markov process that does not have a change in the next distribution state has reached a steady state or equilibrium state. The steady state is denoted by X_L.

A Markov process that has a regular transition matrix will have a steady state.

Example: From the car/bus problem from M-1. Find the steady state distribution.

$$T = \begin{matrix} \\ \text{car} \\ \text{bus} \end{matrix} \begin{bmatrix} \text{car} & \text{bus} \\ 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \qquad X_L = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$T X_L = X_L$$

$$\begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$.7x + .1y = x$$
$$-x \qquad -x$$

$$.3x + .9y = y$$
$$-y \qquad -y$$

$$-.3x + .1y = 0$$
$$.3x - .1y = 0$$

$x + y = 1$ ← all prob adds to 1

userref $x = .25$, $y = .75$

$$X_L = \begin{bmatrix} .25 \\ .75 \end{bmatrix} \begin{matrix} \text{car} \\ \text{bus} \end{matrix}$$

Example: Find the steady state for the pizza problem from M-1.

$$T = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{bmatrix} \text{A} & \text{B} & \text{C} \\ 0.5 & 0.4 & 0.25 \\ 0.3 & 0.2 & 0.5 \\ 0.2 & 0.4 & 0.25 \end{bmatrix} \qquad X_L = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$TX_L = X_L$$

$$\begin{aligned} .5a + .4b + .25c &= a \\ .3a + .2b + .5c &= b \\ .2a + .4b + .25c &= c \\ a + b + c &= 1 \end{aligned}$$

$$\left.\begin{aligned} -.5a + .4b + .25c &= 0 \\ .3a - .8b + .5c &= 0 \\ .2a + .4b - .75c &= 0 \\ a + b + c &= 1 \end{aligned}\right\} \text{rref} \qquad X_L = \begin{bmatrix} 80/201 \\ 65/201 \\ 56/201 \end{bmatrix}$$

Definition: The **limiting matrix** or the **stable matrix** is is defined as $L = \lim_{n \to \infty} T^n$.

Apr 8-11:24 AM