

## Section 5.4: Sinking Funds

The borrower repays the lender by means of a single lump-sum payment at the end of the term of the loan. It is assumed that the borrower makes periodic payments into a fund, called a “sinking fund,” which will accumulate to the amount of the loan to be repaid at the end of the term of the loan. Sometimes the fund is created so that it will be sufficient to repay the loan.

With a sinking fund, interest payments are *usually* paid to the lender in a periodic manner on the entire amount of the loan. These interest payments are sometimes called the service on the loan.

If the rate of interest paid on the loan equals the rate of interest earned on the sinking fund and the sinking fund is designed to exactly pay off the loan (at the end of the term), then the sinking fund method is equivalent to the amortization method.

Example A loan of \$5000 is being repaid in 4 years via the sinking fund method. Write a sinking fund schedule for the repayment of this loan assuming annual payments into the sinking fund. Assume the interest charged on the loan and the interest the sinking fund earns is an annual rate of 6.09%.

$$\begin{aligned} \text{Service} &= 5000 (.0609) \\ &= 304.50 \end{aligned}$$

Payment for the sinking fund.

$$D \frac{1}{s_{\overline{4}|i}} = L = 5000$$

$$D = \frac{L}{s_{\overline{4}|i}} = 1141.43$$

Service

Sinking fund.

Period	Interest Paid	Sinking Fund Deposit	Interest Earned on Sinking Fund	Amount in Sinking Fund	Net amount of the Loan
0	—	—	—	$F_0$ 0	5000
1	304.50	D 1141.43	$I_1 = F_0 i$ 0	$F_1 = P + I_1$ 1141.43	3858.57
2	304.50	1141.43	$I_2 = F_1 \cdot i$ 69.51	2352.37	2647.63
3	304.50	1141.43	143.26	3637.06	1362.94
4	304.50	1141.43	221.50	4999.99	.01

totals  $\underbrace{1218.00 \quad 4565.72 \quad 434.27}$

Total Amnt paid

$$5783.72$$

total paid each period

$$304.50 + 1141.43 = 1445.93$$

Total Interest paid

$$1218.00 - 434.27 = 783.73$$

Because the interest rate were equal

1) The total amount paid in the sinking fund method equals the total amount paid in amortization method.

$$\frac{\text{Amort.}}{\underline{\hspace{2cm}}} = \frac{\text{Sinking fund}}{\underline{\hspace{2cm}}} = \text{Service} + \text{Deposit}$$

$$\frac{L}{a_{\overline{n}|i}} = Li + \frac{L}{s_{\overline{n}|i}}$$

2) Net interest paid in the sinking fund method equals the amount of interest paid in the amortization method.

3) The annual amount added to the sinking fund (deposit and interest) equals the principal repaid in each period of the amortization method.

4) Net amount of the loan in sinking fund method equals the outstanding balance in the amortization method.

Amortization Information

Period	Payment	Interest Paid	Principal Repaid	Outstanding Balance
0	—	—	—	5000
1	1445.93	304.50	1141.43	3858.57
2	1445.93	234.99	1210.94	2647.63
3	1445.93	161.24	1284.69	1362.94
4	1445.93	83.00	1362.93	0.01
Totals	5783.72	783.73	4,999.99	

Sinking fund Information

Period	Interest Paid	Sinking Fund Deposit	Interest Earned	Amount in Fund	Net amount of the Loan
0	—	—	—	0	5000
1	304.50	1141.43	0	1141.43	3858.57
2	304.50	1141.43	69.51	2352.37	2647.63
3	304.50	1141.43	143.26	3637.06	1362.94
4	304.50	1141.43	221.50	4999.99	.01
Total	1218.00	4565.72	434.27		

Sinking Fund Method with Different Interest Rates

Suppose a loan of \$5000 is to be repaid in 4 periods with the sinking fund method.

$i = 10\%$  rate of interest per period paid on loan  
 $j = 6.09\%$  rate of interest earned on sinking fund.

$$\text{Service} = 5000(1.10) = 500$$

Period	Interest Paid	Sinking Fund Deposit	Interest Earned on Sinking Fund	Amount in Sinking Fund	Net amount of the Loan
0	—	—	—	—	5000
1	500	1141.43	0	1141.43	3858.57
2	500	1141.43	69.51	2352.37	2647.63
3	500	1141.43	143.26	3637.06	1362.94
4	500	1141.43	221.50	4999.99	.01

Total 2000

If you wanted an amortization method that was equivalent, what interest rate would you need?

$$\text{Total paid each period} = 500 + 1141.43 = 1641.43$$

$$L = R a_{\overline{n}|k}$$

$$5000 = 1641.43 a_{\overline{4}|k}$$

use TVM solver

$$k = 11.86315\%$$

annual eff.

$$N = 4$$

$$I = \text{solve}$$

$$PV = 5000$$

$$PMT = -1641.43$$

$$FV = 0$$

$$PMT : \text{end.}$$

Example: A payment of  $\$36,000$  is made at the end of each year for 31 years to repay a loan of  $\$400,000$ . If the borrower replaces the capital by means of a sinking fund earning  $3\%$  effective, find the effective rate paid to the lender on the loan.

$\curvearrowright$  Rate for the service on the loan

Sinking fund  
 $L = Ds_{\overline{n}|i}$

$36000$  is Amort. payment

Amort part = service + sinking deposit.

$$36000 = 400000i + \frac{400000}{s_{\overline{31}|3\%}}$$

∴

$$i = 7\%$$

Example: Bob borrows \$5000 for 10 years at 10% convertible quarterly. Bob does not pay interest currently and will pay all accrued interest at the end of 10 years together with the principal. Find the annual sinking fund deposit necessary to liquidate the loan at the end of 10 years if the sinking fund earns 7% convertible semiannually.

at 10 yrs what does Bob owe?

$$5000 \left(1 + \frac{.10}{4}\right)^{40} = 5000 (1.025)^{40}$$

$$5000 (1.025)^{40} = D \overline{s}_{\overline{10}|7.1225\%}$$

$$D = 966.08$$

Loan info  
 $i^{(4)} = 10\%$

$$L = 5000$$

Sinking fund

$$i^{(2)} = 7\%$$

$$n = 10$$

need

$$(1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$i = \left(1 + \frac{.07}{2}\right)^2 - 1$$

$$= 7.1225\%$$

Example: A loan of \$20,000 is to be amortized with monthly payments, at the end of each month, of \$400 until the loan is paid off. The final payment will be some amount less than the monthly payment.  $i^{(12)} = 6\%$

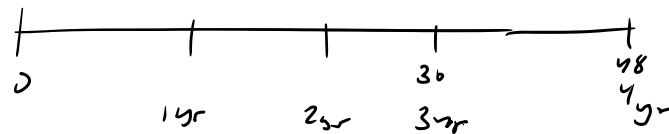
$$i = \frac{i^{(12)}}{12} = \frac{6\%}{12} = \frac{1}{2}\%$$

A) The amount of interest paid in the 4th year.

371.48

used TVM  $\sum$  int

B) The amount of principal paid in the 4th year.



$$P_{37} + P_{38} + \dots + P_{48}$$

4428.52

used TVM  $\sum$  princ