Ch 6 Bonds and other Securities

One of the major applications of the theory of interest is the determination of prices and values for bonds and other securities.

Three main questions of this chapter.

1) Given the desired yield rate of an investor, what price should be paid for a given security?

2) Given the purchase price of a security, what is the resulting yield rate to an investor?

3) What is the value of a security on a given date after it has been purchased?

Section 6.2: Types of Securities

A **security** or **financial instrument** is a tradable asset of any kind: bonds, preferred stocks, common stocks.

Bond

A **bond** is an interest-bearing security which promises to pay a stated amount (or amounts) of money at some future date (or dates). Bonds are commonly used by corporations and governmental units as a means of raising capital.

Terminology for Bonds

The **term** of the bond is the period of time until the bond is redeemed.

The **maturity date** is the end of the term of the bond.

Perpetuals is a bond with an infinite term.

A **Callable bond** is one that may be redeemed early, before the maturity date, at the discretion of the borrower (the bond issuer).

A **Putable bond** is one that may be redeemed early at the discretion of the lender (the bond buyer).

The **redemption date** is any date on which the bond may be redeemed (including the maturity date).

Classification of bonds

Accumulation Bonds: A bond in which the redemption price includes the original loan plus all accumulated interest.

Coupon Bonds: A bond in which periodic payments, called coupons, are made by the bond issuer prior to its redemption.

Indebtedness Issued by the US Treasury

Treasury bills (T-bills): short term debt issued on a discount basis for maturities for 13, 26 or 52 weeks.

Treasury notes: debt of one to seven years.

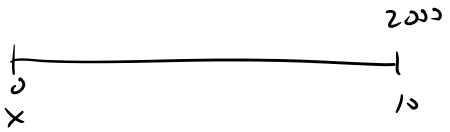
Treasury bonds: long-term debt of seven or more years

Stocks

Preferred stock is a type of security which provides a fixed rate of return, called a **dividend**. While similar to bonds, it differs in that is is an ownership security rather than a debt security. The owner of preferred stock is part owner of the issuing corporation, while the bond owner is a creditor of the corporation.

Common stock is a type of ownership security that does not earn a fixed dividend rate as preferred stock does. Common stock dividends are paid only after interest payments on all bonds and other debt and dividend on preferred stock are paid. The dividend rate is completely flexible.

Example: A 10 year zero-coupon bond has a par value (face value) of \$2000. Find the price that yields an investor 10% effective. (annual eff.)



$X(1+i)^{10} = 2000$

$X = 2000 v^{10} = 2000 (1.10)^{-10} = \$771.09$

Example: A 13-week Treasury bill matures for \$10,000 and is bought at a discount to yield 7.5%. Find the price which must be paid.

Note: T-bills yields are computed as rates of discount rather than rates of interest and are computed on a simple discount basis and typically use bankers rule (actual/360).

13 weeks.
is $13(7) = 91$ days

$$\text{price} = PV = 10000\left(1 - \frac{91}{360}(.075)\right)$$

$$= 9810.42$$

Section 6.3: Price of a Bond

Notation

P = The **price** of a bond (present value)

F = The **par value** or **face amount** of a bond. This is often the amount payable at the maturity date. Bond prices are customarily quoted in terms of a par value of \$100. or 1000] used to find coupons.

C = The **redemption value** of a bond is the amount of money paid at a redemption date to the holder of the bond.
Often C is equal to F. They can differ if the bond matures for an amount not equal to its par value or if the bond is redeemed prior to the maturity date.

r = The **coupon rate** of a bond. The interest rate per coupon payment period.

Fr = The amount of the coupon.

g = The **modified coupon rate** of a bond. The rate g is defined by $Fr = Cg$ or $g = \frac{Fr}{C}$.
g is the coupon rate per unit of redemption value rather than per unit of par value.

i = The **yield rate** of a bond, often called the **yield to maturity**. This is the interest rate actually earned by the investor, assuming the bond is held until it is redeemed or matures. Yield rates are convertible at the same frequency as the coupon rate.

n = The number of coupon payment periods from the date of calculation to the maturity date, or to the redemption date.

K = The present value, computed at the yield rate, of the redemption value at the maturity date or a redemption date. $K = Cv^n$ at a yield rate i.

G = The **base amount** of a bond. The amount G is defined by $Gi = Fr$. Thus G is the amount which if invested at the yield rate i, would produce periodic interest payments equal to the coupons on the bond.

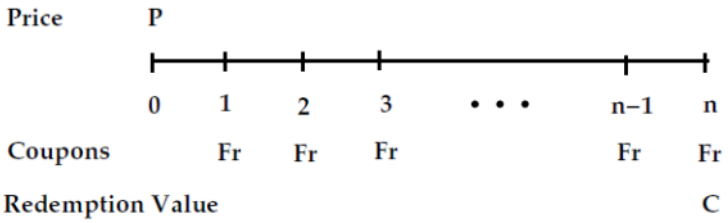
Caution: In everyday business and financial usage, there are three different "yields" associated with a bond.

1. **Nominal yield** is the annualized coupon rate. For example if a \$100 par value bond has coupons totaling \$9 per year, then the nominal yield is 9% per annum.

2. **Current yields** is the ratio of the annualized coupon to the original price of the bond. For example, if the price of the previous bond was \$90, then the current yield is $\frac{9}{90} = 10\%$

3. **Yield to maturity** is the actual annualized yield rate. The level rate of interest earned over the life on the bond reflecting the original price and all payments made by the borrower.

Calculating the Price of a Bond



Basic Formula: $P = Fr\, a_{\overline{n}|} + Cv^n$ price = pv of the coupons + pv of Redemption Amt.

$= Fr\, a_{\overline{n}|} + K$ "Frank"

Premium/Discount Formula:

$P = Fr\, a_{\overline{n}|} + Cv^n$

$= Fr\, a_{\overline{n}|} + C(1 - i\, a_{\overline{n}|})$

$= Fr\, a_{\overline{n}|} + C - Ci\, a_{\overline{n}|}$

$= C + (Fr - Ci)\, a_{\overline{n}|}$

$= C + (C_g - Ci)\, a_{\overline{n}|}$

$P = C + C(g - i)\, a_{\overline{n}|}$

$a_{\overline{n}|} = \frac{1 - v^n}{i}$

$i\, a_{\overline{n}|} = 1 - v^n$

$v^n = 1 - i\, a_{\overline{n}|}$

$C_g = Fr$

Base amount formula: $P = F_r a_{\overline{n}|} + Cv^n = G_i a_{\overline{n}|} + Cv^n$

$F_r = G_i$

$= G(i a_{\overline{n}|}) + Cv^n$

$= G(1 - v^n) + Cv^n$

$= G - Gv^n + Cv^n$

$P = G + (C - G)v^n$

Makeham Formula:

$P = F_r a_{\overline{n}|} + Cv^n = Cv^n + F_r a_{\overline{n}|}$

$F_r = C_g$

$K = Cv^n$

$= K + C_g a_{\overline{n}|}$

$= K + C_g \left(\frac{1 - v^n}{i}\right)$

$= K + \frac{g}{i}(C - Cv^n)$

$P = K + \frac{g}{i}(C - K)$

If $g = i$ Then price $= K + 1(C - K)$

price $= C$

Example: Find the price of a \$1000 par value 10-year bond with coupons at 8.4% payable semiannually, which will be redeemed at \$1050. The bond is bought to yield 10% convertible semiannually.

$F = 1000$

$n = 20$ (total of 20 coupons)

$r = 4.2$

$i = 5$

$C = 1050$

$F_r = 1000(.042) = 42$ semiannual coupon.

$g = \frac{F_r}{C} = \frac{42}{1050} = .04$

$K = C v^n = 1050(1.05)^{-20} = \frac{1050}{(1.05)^{20}}$

$= 395.733957$

$G = \frac{F_r}{i} = \frac{42}{.05} = 840$

Basic $P = F_r a_{\overline{n}|i} + K$

$= 42\, a_{\overline{20}|5\%} + 1050(1.05)^{-20} = 919.15$

TVM solver:
N = 20
I = 5%
PV = solve ← 919.15
PMT = 42
FV = 1050

Premium/discount $P = C + (F_r - Ci)\, a_{\overline{n}|}$

$= 1050 + (42 - 1050(.05))\, a_{\overline{20}|5\%}$

$= 919.15$

Base Amt $P = G + (C - G)v^n$

$= 840 + (1050 - 840)(1.05)^{-20}$

$= 919.15$

Makeham's Formula $P = K + \frac{g}{i}(C - K)$

$= 1050(1.05)^{-20} + \frac{.04}{.05}\left(1050 - 1050(1.05)^{-20}\right)$

$= 919.15$

Example: A 10-year bond, which has just been issued, provides semiannual coupons of 6% a year in arrears (payment scheduled to be paid at the end of the period). The bond is redeemed at par. what price is paid (per \$100 nominal value) if the bond yields an annual effective rate of interest of 8%?

$r = 3\%$

$n = 10(2) = 20$

$F = 100$

$C = F$

annual eff. Rate 8%

$1+i = \left(1+\frac{i^{(2)}}{2}\right)^2$

$1.08 = \left(1+\frac{i^{(2)}}{2}\right)^2$

$j = \frac{i^{(2)}}{2} = (1.08)^{1/2} - 1$

$Fr = 100(.03) = 3$ per semiannual period

$\text{price} = 3a_{\overline{20}|j} + 100v^{20}$

$= 3a_{\overline{20}|j} + 100(1+j)^{-20}$

$= 87.37$

$3a_{\overline{20}|j} + 100(1.08)^{-10}$

by 4.1/4.2 methods.

$\text{price} = 6a_{\overline{10}|i}^{(2)} + 100(1.08)^{-10}$

6 = total coupons paid in the yr.

Example: Two \$1000 bonds redeemable at par at the end of the same period are bought to yield 4% convertible semiannually. One bond costs \$1136.78 and has a coupon rate of 5% payable semiannually. the other bond has a coupon rate of 2.5% payable semiannually. Find the Price of the second bond.

$F = C = 1000$ $\quad i^{(2)} = 4\%$ $\quad \frac{i^{(2)}}{2} = 2\%$

Bond 1 $\quad F = C = 1000$

Price = 1136.78

$r = \frac{.05}{2} = .025$

$n = ?$

$Fr = 1000(.025) = 25$

$P = C + (Fr - Ci)\, a_{\overline{n}|}$

$1136.78 = 1000 + (25 - 1000(.02))\, a_{\overline{n}|}$

$136.78 = 5\, a_{\overline{n}|}$

$\frac{136.78}{5} = a_{\overline{n}|}$

Bond 2

price = solve

$n = ?$

$r = \frac{.025}{2} = .0125$

$Fr = 12.5$

$P = 1000 + (12.5 - 1000(.02))\, a_{\overline{n}|}$

$P = 1000 + (12.5 - 20)\, \frac{136.78}{5}$

$P = 794.83$

Bond if $F = C \Rightarrow r = g$ since $F_r = C_g$

If $Price = P = C$

$$C = K + \frac{g}{i}(C - K)$$

$$C - K = \frac{g}{i}(C - K)$$

$$1 = \frac{g}{i} \Rightarrow i = g$$

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if $Price = C = F \Rightarrow i = g = r \Rightarrow i = r$