Section 6.4: Premium and Discount

With bonds, the price P of the bond is compared to the redemption value C.

- If $P > C$, then the bond is said to sell at a **premium**, and $P - C$ is the amount of the premium.
- If $P < C$, then the bond is said to sell at a **discount**, and $C - P$ is the amount of discount.

The amount of premium and amount of discount can be expressed using the Premium/Discount formula.

Premium/Discount formula: $P = C + (Fr - Ci)a_{\overline{n}|}$

Amount of Premium:

$\boxed{P-C}$

$$P - C = (Fr - Ci)a_{\overline{n}|}$$
$$P - C = (Cg - Ci)a_{\overline{n}|}$$
$$P - C = C(g - i)a_{\overline{n}|}$$

$g > i$

Amount of discount:

$C - P$

$$C - P = C(i - g)a_{\overline{n}|}$$

$g < i$ or $i > g$

The purchase price is usually different from the redemption value, thus there will be a profit(equal to the discount) or a loss(equal to the premium) at the redemption date.

(in terms of the Redemption amt)

As a result of this profit/loss at the redemption date, the amount of each coupon should not be considered as interest income to an investor. Each coupon will be divided into interest earned and principal adjustments.

When this approach is used, the value of the bond will be continually adjusted from the price on the purchase date to the redemption value on the redemption date. These adjusted values of the bond are called the **book values**.

When the bond is bought at a premium, the book value will gradually be adjusted downward. This process is called **amortization of the premium** or **writing down**. The principal adjustment amount is often called the "amount for amortization of premium."

When the bond is bought at a discount, the book value will gradually be adjusted upward. This process is called **accumulation of discount** or **writing up**. The principal adjustment amount is often called the "amount for accumulation of discount."

Example: Write a bond amortization schedule for a $1000 par value two-year bond with 8% coupons paid semiannually bought to yield 6% convertible semiannually.

find price $F = C = 1000$

$n = 4$ $r = \frac{8\%}{2} = 4\%$

$i = \frac{6\%}{2} = 3\%$

$Fr = 1000(.04) = 40$

$P = Fr\, a_{\overline{n}|} + Cv^n$

$= 1000(.04)\, a_{\overline{4}|3\%} + 1000v^4$

$= 1037.17$

$N = 4$
$I = 3\%$
$PV =$ solve
$PMT = Fr$
$FV = 1000$

Bond sold at a premium

premium $= 37.17$

principal adjustment

Period	Coupon	Interest Earned	Amount for amort. of premium	Book Value	
0				1037.17	B_0
1	40	$I_1 = B_0 \cdot i$ $1037.17(.03) = 31.12$	$P_1 = Fr - I_1$ 8.88	1028.29	$B_1 = B_0 - P_1$
2	40	30.85	9.15	1019.14	B_2
3	40	30.57	9.43	1009.71	B_3
4	40	30.29	9.71	1000	B_4
Total	160	122.83	37.17		

$160 = 122.83 + 37.17$

$B_1 = B_0(1+i) - Fr$

amt of premium $= 8.88\, s_{\overline{4}|3\%}$

Period	Coupon	Interest Earned	Amount for amort. of premium principal adj.	Book Value
0	—	—	—	1037.17
1	40	31.12	8.88	1028.29
2	40	30.85	9.15	1019.14
3	40	30.57	9.43	1009.71
4	40	30.29	9.71	1000
Total	160	122.83	37.17	

<u>In the bond amortization schedule</u>:

- The principal adjustment forms a geometric progression with common ratio $1 + i$.
- The Book values correspond to the value(price) of the bond immediately after the coupon payment at the original yield rate.
- The sum of the principal adjustment column is equal to the premium or discount.
- The sum of the interest earned column is equal to the difference between the sum of the coupons and the sum of the principal adjustment column.
- $B_{t+1} = B_t(1 + i) - Fr$
- A general table can be found on page 208 of the textbook.

$Fr = Cg$

Since $F = C \rightarrow r = g$

Example: Write a bond amortization schedule for a $1000 par value two-year bond with 8% coupons paid semiannually bought to yield 10% convertible semiannually.

$F = C = 1000$

$r = 4\%$

$i = 5\%$

$Fr = 1000(4\%)$
$= 40$

$P = Fra_{\overline{n}|} + Cv^n$
$= 40\, a_{\overline{4}|5\%} + 1000v^4$
$= 964.54$

$i = 5\% \quad g = 4\%$

$i > g$

Sell at a discount

Amt of discount is
$1000 - 964.54$
$= 35.46$

principal adjustment

Period	Coupon	Interest Earned	Amt. for accumulation of discount	Book Value
0				B_0 964.54
1	40	$I_1 = B_0 \cdot i$ 48.23	$P_1 = Fr - I_1$ 8.23	B_1 972.77
2	40	48.64	8.64	B_2 981.41
3	40	49.07	9.07	B_3 990.48
4	40	49.52	9.52	B_4 1000
Total	160	195.46	35.46	

$B_1 = B_0 - P_1$

we "understand" that these values are negative.

Example: A 100 par value bond with 10% semiannual coupons is purchased to yield 15% convertible semiannual. If the bond was issued on June 1, 1990, with the first coupon paid on Dec 1, 1990, and has a maturity date of June 1, 2010, find the book value of the bond on June 1, 2000, just after the coupon is paid.

Started 6/1/1990

ends 6/1/2010

find price at 6/1/2000

#of coupons still remain:

$n = 20$

dec 1 2000 — 1 coupon

2 coupons $\left\{ \begin{array}{c} 2001 \\ \downarrow \\ 2009 \end{array} \right\}$ 18 coupon

1 2010 — 1 coupon

20 coupons

$F = 100 \qquad C = 100 \qquad i = \frac{15}{2}\% = 7.5\%$

$r = \frac{10}{2}\% = 5\%$

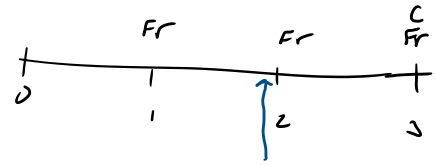
$$\text{price} = 100(.05)\, a_{\overline{20|}\,7.5\%} + 100\, v^{20}$$

$$= 74.5138$$

N = 20 I% = 7.5 PV = solve → 74.5138

PMT = 5 FV = 100

Section 6.5: Valuation Between Coupon Payment Dates



In the preceding section we found that the book value of a bond on consecutive coupon dates had the relationship of

$$B_{t+1} = B_t(1+i) - Fr$$

Our goal is to calculate the book value between coupon dates: B_{t+k} where $0 < k < 1$.

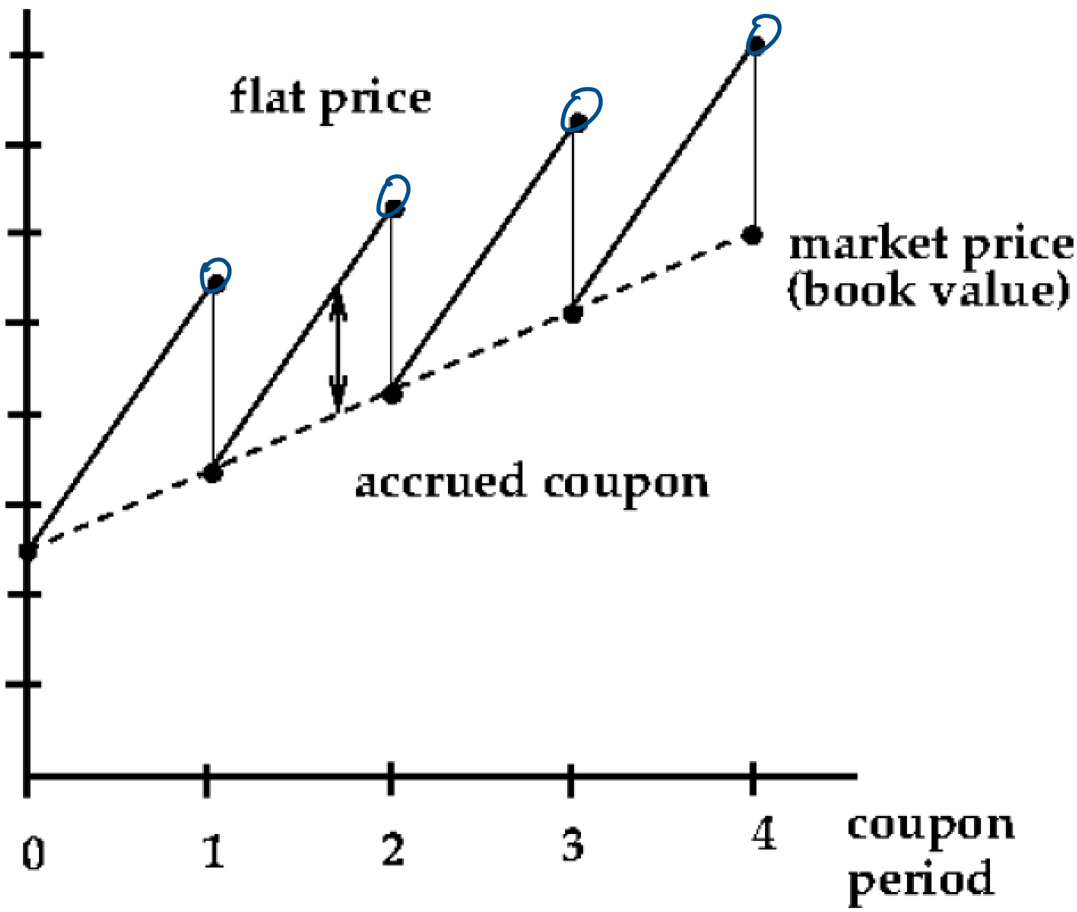
When a bond is bought between coupon payment dates, it is necessary to split the coupon for the current period between the prior owner and the new owner.

Since the new owner will receive the entire coupon at the end of the period, the purchase price should include a payment to the prior owner for the portion of the coupon corresponding to the portion of the period the prior owner had the bond. This payment is called the **accrued coupon**. Notation: Fr_k.

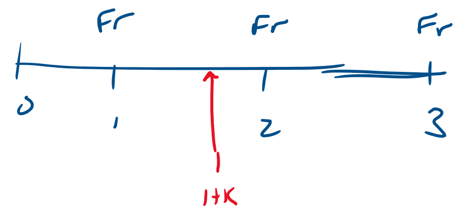
The **flat price** of a bond is the amount of money which actually change hands on the date of the sale. Also called the "full price", "purchase price", "price plus accrued", "dirty price". Notation: B^f_{t+k}

The **market price** of a bond as the price excluding the accrued coupon. Also called the "clean price" Notation: B^m_{t+k}

Thus for $0 < k < 1$, $$B^f_{t+k} = B^m_{t+k} + Fr_k$$



Pg 3: Theoretical Method



Methods for Bond Valuation Between Coupon Dates

Theoretical Method is based on compound interest.

The flat price on the interim date is the value on the preceding coupon date accumulated with compound interest at the yield rate for the fractional period.

$$B^f_{t+k} = B_t(1+i)^k$$

The accrued coupon is computed by $\quad Fr_k = Fr\boxed{\left[\frac{(1+i)^k - 1}{i}\right]}$

Section 3.6

unknown time section

The market price is then the difference of the flat price and the accrued coupon.

$$B^m_{t+k} = B^f_{t+k} - Fr_k = B_t(1+i)^k - Fr\left[\frac{(1+i)^k - 1}{i}\right]$$

Pg 4: Practical Method

Practical Method is based on simple interest.

The flat price is computed computed by $B_{t+k}^{f} = B_t(1 + ki)$

The accrued coupon is computed as the pro rata portion of the coupon, i.e. is proportional to the amount of the coupon period that has passed.

$$Fr_k = kFr$$

The market price is then the difference of the flat price and the accrued coupon.

$$B_{t+k}^{m} = B_{t+k}^{f} - Fr_k = B_t(1 + ki) - kFr$$

Pg 5: Semi-Theoretical Method

Semi-Theoretical Method is a combination of compound interest and pro rata accrued coupon.

$$B^f_{t+k} = B_t(1+i)^k \qquad Fr_k = kFr \qquad B^m_{t+k} = B^f_{t+k} - Fr_k = B_t(1+i)^k - kFr$$

Computing the value of $k = \dfrac{\text{number of days since last coupon paid}}{\text{number of days in coupon period}}$

- Corporate bonds use 30/360 method. (ordinary)
- Government bonds and notes use actual/actual method.

- If not told what type of bond(government/corporate), assume government.
- If no method is given when computing the value, assume semi-theoretical.

$Fr = Cg \quad F = C \Rightarrow r = g$

$g < i \quad$ discount.

Example: A 100 par value bond with 10% semiannual coupons is purchased to yield 15% convertible semiannual. If the bond was issued on June 1, 1990, with the first coupon paid on Dec 1, 1990, and has a maturity date of June 1, 2010, find the book value of the bond on August 10, 2002. Use all three methods.

$F = 100 = C$

$r = 5\% = g$

$i = 7.5\%$

$Fr = 100(.05) = 5$

Start June 1, 1990 → coupons Dec 1, June 1

mature at June 1, 2010

length 20 yrs. → 40 coupons.

find the value of the Bond on Aug 10, 2002

we need the Book value (price) on June 1, 2002

June 1, 2002 → June 1, 2010 ⇒ 8 yrs → 16 coupons left.

$$\text{price} = 5\, a_{\overline{16}|7.5\%} + 100 v^{16}$$

$N = 16$

$I = 7.5\%$

$PV = $ solve

$PMT = 5$

$FV = 100$

$$\text{price} = B = 77.1462315$$

need to find k

$$k = \frac{dbd(6.0102,\ 8.1002,\ \text{ACT})}{dbd(6.0102,\ 12.0102,\ \text{ACT})} = \frac{70}{183}$$

Theoretical method.

$$B^f = B(1+i)^k = (77.1462315)(1+.075)^{70/183}$$

$$= 79.3107207$$

$$Fr_k = Fr\left[\frac{(1+i)^k - 1}{i}\right] = 5\left(\frac{(1.075)^{70/183} - 1}{.075}\right) = 1.869988$$

$$B^m = B^f - Fr_k = 77.44018$$

Practical method.

$$B^f = B(1+\kappa i) = (77.14623)\left(1+\frac{70}{183}(.075)\right)$$

$$= 79.35944$$

$$Fr_\kappa = \kappa \cdot Fr = \frac{70}{183}(5) = 1.912568$$

$$B^m = B^f - Fr_\kappa = 77.44687$$

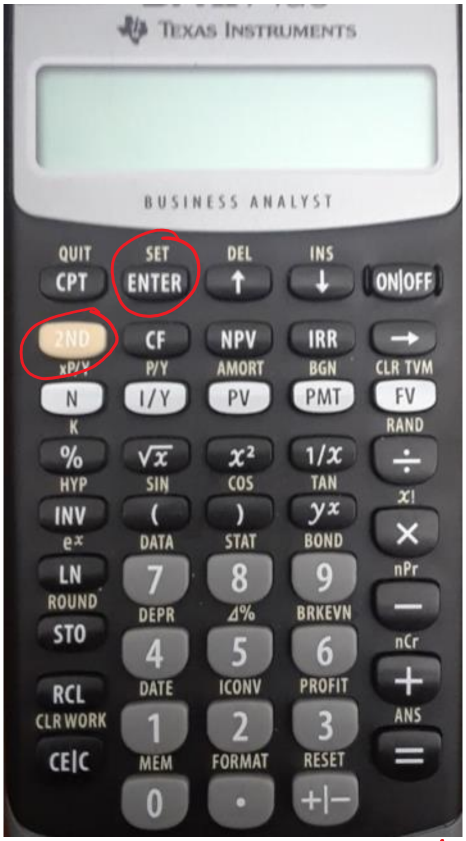
Semi Theoretical

$$B^f = B(1+i)^\kappa = 79.31017201$$

$$Fr_\kappa = \kappa Fr = 1.912568$$

$$B^m = B^f - Fr_\kappa = 77.39757527$$

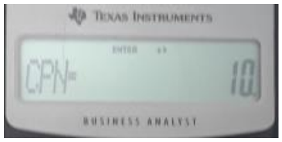
Bond calculator



Bond work sheet

$\boxed{2^{nd}}$ $\boxed{9}$

settelment date.

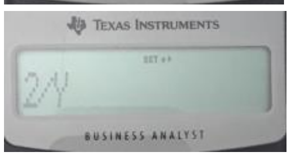


nominal coupon (whole year)

Redemption date

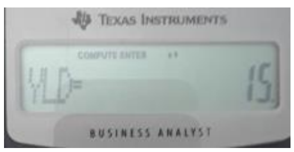
Redemption value

Act / 360



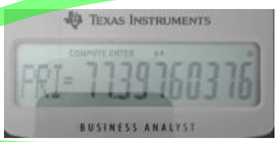
# of coupons per yr.

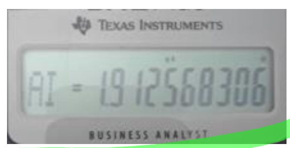
1/Y or 2/Y



nominal yield

uses Semi theoretical method.





price

accrued coupon Frk

Example: A \$1000 bond with semiannual coupons at $i^{(2)} = 6\%$ matures at par on October 15, 2035. The bond is purchased on June 28, 2020 to yield the investor $i^{(2)} = 7\%$. What is the purchase price? Assume simle interest between bond coupon dates and use an exact day count.

practical

actual/Actual.

Coupons paid.

April 15 and oct 15

$F = 1000 = C$

matures on Oct 15, 2035 :

$r = 3\%$

Purchase on 6/28/2020

$i = 3.5\%$

$Fr = 1000(.03) = 30$

2020 – 1 coupon

2021 } 15
2035 }

×2

30

find Book value on April 15, 2020

still have 31 coupons.

$$B = \text{price} = 30\, a_{\overline{31}|} + 1000 v^{31} = 906.3186$$

$$K = \frac{dbd(4.15\,20,\ 6.28\,20,\ ACT)}{dbd(4.15\,20,\ 10.15\,20,\ ACT)} = \frac{74}{183}$$

$$B^f = B(1 + iK) = 906.3186\left(1 + (.035)\left(\frac{74}{183}\right)\right) = 919.1457$$

purchase price.

$$Fr_K = \frac{74}{183}(30) = 12.131247$$

$$B^m = B^f - Fr_K = 907.01446$$