Section 6.7: Callable and Puttable Bonds

A **Callable bond** is a bond for which the issuer (borrower) has an option to redeem prior to the normal maturity date. The earliest date is the **call date**.

A **Putable bond** (or a put bond) is a bond for which the owner (lender) has an option to redeem prior to the normal maturity date. The earliest date is the **put date**.

Most bonds issued by corporations and by state/local governmental entities are callable.
Bonds issued by the US Treasury are generally not callable.
Putable bonds are much less common.

A callable bond will sell at a higher yield rate (lower price) than otherwise identical non-callable bond because of the uncertainty attached to the ultimate term of the bond.

Reasons why a borrower would issue a bond on which they will receive a lower price and/or pay a higher coupon rate.

- Flexibility to pay off the indebtedness early and avoid paying the coupon after the bond is called.
- Interest rate may decline. They may call one issue of bonds early and then refinance the indebtedness at a lower rate.

A putable bond will sell at a lower yiled rate(higher price) than an otherwise identical non-putable bond.

Question: How do we calculate the price of a callable bond if the term of the bond is uncertain?

Answer: The lender should assume that the borrower will exercise the call option to the disadvantage of the lender and prices should be calculated accordingly.

Callable Bond:

If the redemption values on all redemption dates are equal.

1) If the bond sells at a premium $(g > i)$, assume that the redemption date will be the earliest possible date.
   Selling at a premium means there is a loss at redemption. The most unfavorable situation to the lender occurs when the loss is as early as possible.

2) If the bond sells at a discount $(i > g)$, assume that the redemption date will be the latest possible date.
   Selling at a discount means there is a gain at redemption. The most unfavorable situation to the lender occurs when the gain is as late as possible.

If the redemption values on all redemption dates are not equal.

The bond holder should compute the purchase price for all possible call dates at a desired yield rate and then pay no more than the lowest of these prices to guarantee the minimum desired yield rate.

The rules are reversed for putable bonds. The most unfavorable put date is the one that produces the largest purchase price at the yield rate.

Example: Consider a 10 year 100 par value 6% bond with semiannual coupons callable at ~~start~~ end of year 5, 6, 7, 8, 9, and 10 (after the coupon payment). Assume callable at par value and a nominal yield of 2% convertible semiannually.

Compute the price of the bond at all callable times.

$F = C = 100$ $i = 1\%$

$r = 3\%$

Redemption Date	purchase price
n=	
10 (~~start~~ end of year 5)	118.94261
12 (~~start~~ end of year 6)	122.51015
14 (~~start~~ end of year 7)	126.00741
16 (~~start~~ end of year 8)	129.43575
18 (~~start~~ end of year 9)	132.79654
20 (~~start~~ end of year 10)	136.09111

called after 10 coupons

$\text{price} = 3\, a_{\overline{10}|1\%} + 100 v^{10} = 118.94261$

after 12 coupons

$P = 3\, a_{\overline{12}|1\%} + 100 v^{12} = 122.51015$

N = ___

I% = 1%

PV = solve

PMT = 3

FV = 100

Example: Consider a 10 year 100 par value 6% bond with semiannual coupons callable at ~~start~~ end of year 5, 6, 7, 8, 9, and 10 (after the coupon payment). Assume callable at par value and a nominal yield of 2% convertible semiannually.

Suppose the bond is not called, using the price computed, what is the yield to maturity.

Redemption Date	purchase price	yield to maturity for purchase price
10 (~~start~~ end of year 5)	118.9426	3.714824%
12 (~~start~~ end of year 6)	122.51015	3.33434%
14 (~~start~~ end of year 7)	126.00741	2.97421%
16 (~~start~~ end of year 8)	129.43575	2.63274%
18 (~~start~~ end of year 9)	132.79654	2.30844%
20 (~~start~~ end of year 10)	136.09111	2.000%

$N = 10$

$I = \text{solve} \rightarrow \frac{i^{(2)}}{2} = \frac{3.714824\%}{2}$

$PV = -118.9426$

$PMT = 3$

$FV = 100$

$i^{(2)} = 3.714824\%$

Example: Consider a 10 year 100 par value 6% bond with semiannual coupons callable at ~~start~~ end of year 5, 6, 7, 8, 9, and 10 (after the coupon payment). Assume callable at par value and a nominal yield of 2% convertible semiannually.

The table shows the different yields for the different times that the bond could be called.

Redemption Date	purchase price	yield to maturity for purchase price	yield to call for price of 136.0911	yield to call for price of 118.94261
10 (~~start~~ end of year 5)	118.9426	3.714824%	−1.01774%	2.00000%
12 (~~start~~ end of year 6)	122.51015	3.33434%	-0.01270%	2.57274%
14 (~~start~~ end of year 7)	126.00741	2.97421%	0.70649%	2.98165%
16 (~~start~~ end of year 8)	129.43575	2.63274%	1.24600%	3.28784%
18 (~~start~~ end of year 9)	132.79654	2.30844%	1.66523%	3.52539%
20 (start of year 10)	136.09111	2.00000%	2.00000%	3.71482%

Example: Consider a 10 year 100 par value 6% bond with semiannual coupons callable at ~~start~~ end of year 5, 6, 7, 8, 9, and 10 (after the coupon payment). Assume callable at par value and a nominal yield of 7% convertible semiannually.

$F=100=C$

$n=20$ total # of coupons

$g=r=3\%$

$i=3.5\%$

$Fr=Cg$

Since $F=C \Rightarrow r=g$

$i>g$

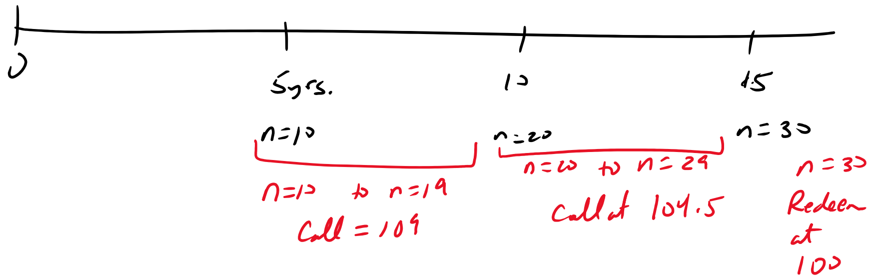
discount

Redemption Date	purchase price	yield to maturity for purchase price	yield to call for price of 92.89380	yield to call for price 95.84170
10 (~~start~~ end of year 5)	95.84170	6.57394%	7.74098%	7.00000%
12 (~~start~~ end of year 6)	95.16833	6.66981%	7.49213%	6.85702%
14 (~~start~~ end of year 7)	94.53974	6.76006%	7.31524%	6.75532%
16 (~~start~~ end of year 8)	93.95294	6.84500%	7.18325%	6.67941%
18 (~~start~~ end of year 9)	93.40516	6.92489%	7.08117%	6.62067%
20 (~~start~~ end of year 10)	92.89380	7.00000%	7.00000%	6.57394%

Example: Consider a \$100 par value 4% bond with semiannual coupons called at \$109 on any coupon date starting 5 years after issue for the next 5 years, at \$104.50 starting 10 years after issue for the next 5 years, and maturing at \$100 at the end of 15 years. Find the highest price which an investor can pay and still be certain of a yield of

(a) 5% convertible semiannually.

~~(b) 3% convertible semiannually.~~



$F = 100 \qquad r = 2\%$

discount / premium?

$i = 2.5\% = .025$

find g)

$C = \$109$

$Fr = Cg$

$100(.02) = 109g$

$g = \frac{2}{109} = .018349$

$i > g$ discount

use $n = 19$

$C = 104.5$

$g = \frac{Fr}{C} = \frac{2}{104.5}$

$= .0191$

$i > g$ discount

$n = 29$

$C = 100$

$g = r = .02$

$i > g$ discounts

$n = 30$

find price

$n = 19$

TVM
N = 19
I = 2.5
PV = solve
PMT = 2
FV = 109

$P = 98.14$

$n = 29$

TVM
N = 29
I = 2.5
PV = solve
PMT = 2
FV = 104.5

$P = 91.97$

TVM
N = 30
I = 2.5
PV = solve
PMT = 2
FV = 100

$P = 89.53$

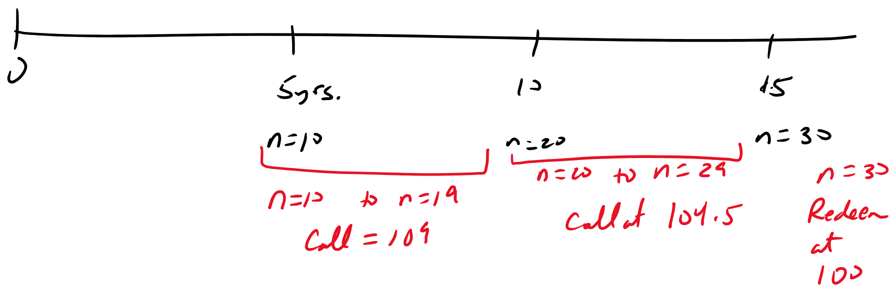
Answer is \$89.53

Example: Consider a \$100 par value 4% bond with semiannual coupons called at \$109 on any coupon date starting 5 years after issue for the next 5 years, at \$104.50 starting 10 years after issue for the next 5 years, and maturing at \$100 at the end of 15 years. Find the highest price which an investor can pay and still be certain of a yield of

~~(a) 5% convertible semiannually.~~
(b) 3% convertible semiannually.

$F = 100 \qquad r = 2\%$

$i = \frac{3}{2}\% = 1.5\% = .015$



find g)

$C = \$109$

$Fr = Cg$

$100(.02) = 109g$

$g = \frac{2}{109} = .018349$

$g > i$

premium

$n = 10$

$C = 104.5$

$g = \frac{Fr}{C} = \frac{2}{104.5}$

$= .0191$

$g > i$

premium

$n = 20$

$C = 100$

$g = r = .02$

$g > i$

premium

$n = 30$

find price

$n = 10 \rightarrow$ price $= 112.37$

$n = 20 \rightarrow$ price $= 111.9245$ ← Answer

$n = 30 \rightarrow$ price $= 112.01$

Section 10: Other Securities

Preferred Stocks and perpetual bonds are both types of fixed income securities without fixed redemption dates.

Trate the dividens as a perpetuity Imediate.

$$\text{price} = \frac{Fr}{i} = \frac{D}{i}$$

geometric

$$\text{price} = \frac{Fr}{i-k}$$

arithmetic

$$\text{price} = \frac{Fr}{i} + \frac{Q}{i^2}$$

Example: A preferred stock pays \$10 dividend at the end of the first year, with each successive dividend being 5% greater than the preceding one. What level annual dividend would be equivalent if $i = 12\%$?

$i = 12\%$ $k = 5\%$

geometric

$$\text{Price} = \frac{10}{.12 - .05} = \frac{10}{.07}$$

level dividend

$$\text{price} = \frac{D}{.12}$$

$$\frac{10}{.07} = \frac{D}{.12} \longrightarrow D = \frac{(.12)(10)}{.07} = 17.1429$$

Common Stocks are not fixed income securities. However, theoretical prices may be computed by assuming the dividends are either fixed or future dividend payments follow a geometric progression.

Example: A common stock has a dividend of \$2 per share paid at the end of the year. Assume the dividend is fixed for 10 years and then grows at 2% per year forever. Find the theoretical price to earn an investor an annual effective yield of 7%.

$$\text{price} = 2a_{\overline{10}|7\%} + \left(\frac{2(1.02)}{\underbrace{.07-.02}_{i-k}}\right) v^{10}$$

$$= 34.788$$