Section 3.1: Limits

A limit is way to discuss the values/properties of a function(y-values) as the x-values get either get close to (but not equal) a number or procede in a particular direction. There are three forms to the limit.

$\lim_{x \to a^-} f(x)$ $\lim_{x \to a^+} f(x)$ $\lim_{x \to a} f(x)$

Graphical Limits

Example: Use the graph to answer these questions.

$\lim_{x \to -3^-} r(x) =$ $\lim_{x \to -3^+} r(x) =$

$\lim_{x \to -3} r(x) =$ $r(-3) =$

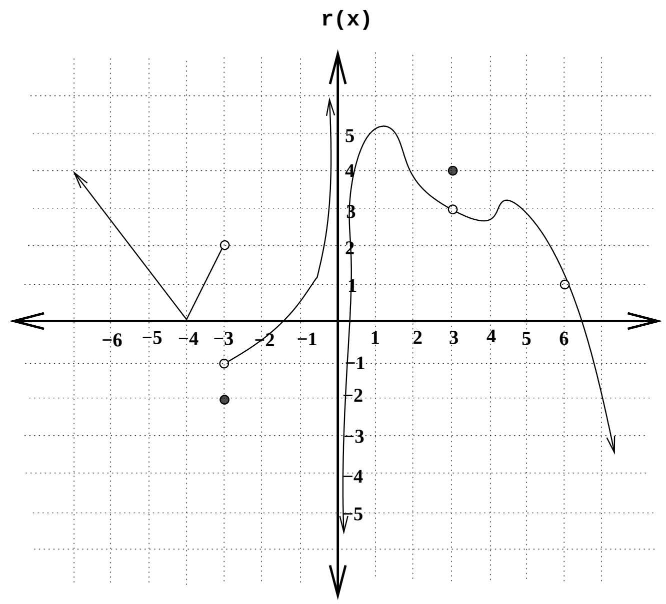
$\lim_{x \to 3^-} r(x) =$ $\lim_{x \to 3^+} r(x) =$

$\lim_{x \to 3} r(x) =$ $\lim_{x \to -5} r(x) =$

$\lim_{x \to 0^+} r(x) =$ $\lim_{x \to 0^-} r(x) =$

$\lim_{x \to 0} r(x) =$

$\lim_{x \to \infty} r(x) =$ $\lim_{x \to -\infty} r(x) =$



Non-Graphical Limits

Example: $\lim_{x \to 2} 3x^2 + 5 =$

x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
f(x)					XXXX				

Example: $\lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1} =$

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
f(x)					XXXX				

The algebraic method of evaluating the $\lim_{x \to a} f(x)$ is to compute $f(a)$. The final answer is based on these cases.

Case 1: $f(a) = L$, then the answer is L.

Case 2: $f(a) = \frac{0}{L}$ with $L \neq 0$, then the answer is 0.

Case 3: $f(a) = \frac{0}{0}$, then simplify and try again.

Case 4: $f(a) = \frac{L}{0}$ with $L \neq 0$, then the answer is either $-\infty$, ∞, or DNE.

Example: Compute these limits.

A) $\lim_{x \to 5} -x^2 + 35 =$

B) $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 7x + 12} =$

C) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} =$

D) $f(x) = \begin{cases} 2x^3 + 4x - 4 & \text{if } x \geq 1 \\ x^4 + x^2 & \text{if } x < 1 \end{cases}$

$\lim_{x \to 0} f(x) =$

$\lim_{x \to 1} f(x) =$

E) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 4x - 5} =$

Example: If $f(x) = x^2 + 2x$, compute $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$.