

**Section 3.3: More Limits and other Information**

Example: Compute these limits.

A)  $\lim_{x \rightarrow 3^+} \frac{2}{x-3} =$

C)  $\lim_{x \rightarrow 3} \frac{2}{x-3} =$

B)  $\lim_{x \rightarrow 3^-} \frac{2}{x-3} =$

D)  $\lim_{x \rightarrow 4} \frac{-2}{(x-4)^2} =$

**Definition:**  $x = a$  is said to be a vertical asymptote of the function  $f(x)$  provided

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Example: Find the vertical asymptotes and the holes of these functions.

A)  $g(x) = \frac{x^2 - 4}{x^2 - 7x + 10}$

B)  $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4}$

C)  $m(x) = \frac{x(x+1)}{x^2(x+4)}$ 

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**End behavior of functions and limits**

Example: Compute these limits.

A)  $\lim_{x \rightarrow \infty} 3x^2 - 5x + 8 =$

B)  $\lim_{x \rightarrow -\infty} -2x^2 + 6x + 1 =$

C)  $\lim_{x \rightarrow -\infty} 7x^6 + 5x^4 + 3x^2 + 1 =$

D)  $\lim_{x \rightarrow \infty} 3x^5 - 6x^8 + 2x + 4 =$

E)  $\lim_{x \rightarrow -\infty} -2x^5 + 1 =$

F)  $\lim_{x \rightarrow \infty} \frac{3x^4 + 2x + 7}{5x^4 + 7x + 5} =$

**Definition:**  $y = a$  is said to be a horizontal asymptote of the function  $f(x)$  provided

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Example: Compute these limits.

$$\text{A) } \lim_{x \rightarrow \infty} \frac{3x^4 - 2x^2 + 7}{x^5 + 2x + 1} =$$

$$\text{B) } \lim_{x \rightarrow \infty} \frac{3x^4 - 2x^6 + 7}{7x^6 + 3x^2 + 8} =$$

$$\text{C) } \lim_{x \rightarrow \infty} \frac{3x^6 + 2x + 7}{-4x^5 + 7x + 5} =$$

$$\text{D) } \lim_{x \rightarrow \infty} \frac{6x^4 + 3x^5 + 1}{7 + 2x^2} =$$

Example: Give the horizontal asymptote of these functions.

$$\text{A) } f(x) = \frac{3x^4 - 2x^2 + 7}{x^5 + 2x + 1}$$

$$\text{B) } f(x) = \frac{3x^4 - 2x^6 + 7}{7x^6 + 3x^2 + 8}$$

$$\text{C) } f(x) = \frac{3x^6 + 2x + 7}{-4x^5 + 7x + 5}$$

Example: Find the horizontal and the vertical asymptotes of this functions.

$$y = \frac{7(2x + 7)(4 - 5x)}{(6x + 2)(3x - 11)}$$


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### Exponential functions and limits

Example: Compute the following limits.

$$\text{A) } \lim_{x \rightarrow \infty} e^{3x} =$$

$$\text{B) } \lim_{x \rightarrow -\infty} e^{3x} =$$

$$\text{C) } \lim_{x \rightarrow \infty} \frac{5e^{6x} + 3}{2 + 4e^{6x}} =$$

$$D) \lim_{x \rightarrow -\infty} \frac{5e^{6x} + 3}{2 + 4e^{6x}} =$$

$$E) \lim_{x \rightarrow \infty} \frac{6e^{-x} + 2}{5e^{-x} + 7} =$$

$$F) \lim_{x \rightarrow -\infty} \frac{6e^{-x} + 2}{5e^{-x} + 7} =$$

$$G) \lim_{x \rightarrow \infty} \frac{3^x + 7}{4^x + 7} =$$