

### Section 3.4: The Derivative

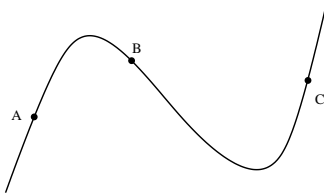
Suggested problems: 3, 7, 17, 21, 25, 29, 31-38, 47, 48, 60a, 60c, 61a, 61c, 63b

#### Average Rate of Change

Example: The revenue (in dollars) for selling  $x$  widgets is given by  $R(x) = 40x - 0.2x^2$ .

- A) What is the change in revenue if production is changed from 30 items to 110 items?
- B) What is the average change(average rate of change) in revenue for this change in production?
- C) What is the average change in revenue when production goes from 80 items to 160 items?

Example: Which pair of points has an average rate of change that is the largest? the smallest?

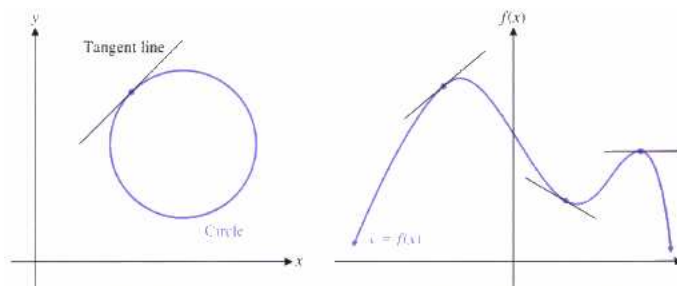



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#### Instantaneous Rate of Change

Definition: The instantaneous rate of change of a function  $f(x)$  at  $x = a$  is the slope of the tangent line at  $x = a$  and is denoted  $f'(a)$ .

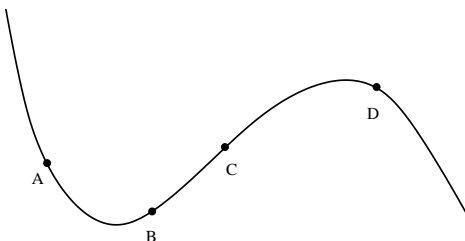
#### Graphical Method



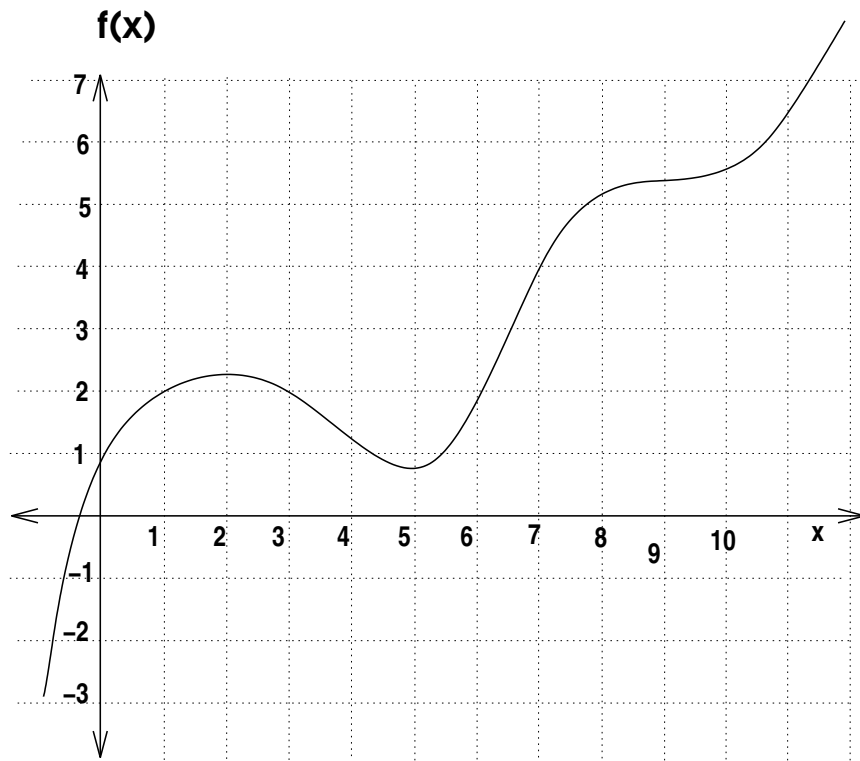
(A) Tangent line for a circle

(B) Tangent lines for the graph of a function

Example: At which point is the instantaneous rate of change the greatest? the least?



Example: Use this graph to answer these questions.



- A) Estimate the instantaneous rate of change at  $x = 1$  and  $x = 7$ .
- B) Find the equation of the tangent line at  $x = 1$  and  $x = 7$ .
- C) At what values of  $x$  does  $f(x)$  have an instantaneous rate of change of 0?

### Algebraic Method

Example: Compute the following average rates of change from  $x = 2$  to  $x = a$  for the function  $f(x) = -x^2 + 10x$

$a$	5	3	2.4	2.1	2.01
$\frac{f(a) - f(2)}{a - 2}$					

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Definition: For  $f(x)$ , the instantaneous rate of change at  $x = a$  (or rate of change at  $x = a$ ) is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

If this limit exists, then we say that the function,  $f(x)$  is differentiable at  $x = a$ . If  $f'(a)$  exists for all  $b < a < c$ , then we say that  $f(x)$  is differentiable on the interval  $(b, c)$ .

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Example: The revenue (in dollars) for selling  $x$  widgets is given by  $R(x) = 40x - 0.2x^2$ .

A) Find the instantaneous rate of change when 75 items are sold.

B) Use the answer/work for part A to estimate  $R'(76)$ .

C) Explain the meaning for  $R'(75)$ .

Example: Find  $f'(3)$  for  $f(x) = 3 + \frac{2}{x}$

Example: You have been told that the derivative for  $f(x) = 3 + \frac{2}{x}$ , is  $f'(x) = \frac{-2}{x^2}$ . Find the equation of the tangent line at  $x = 2$  and  $x = 0$ .

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Where does the derivative of a function not exist?

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Example: Find  $f'(4)$  for  $f(x) = 6 + 2\sqrt{x}$