Section 3.7: Marginal Analysis

Definition: In applications, the word **marginal** is used to refer to the derivative. Thus marginal cost refers to the derivative of the cost function, $C'(x)$.

Example: A company knows that manufacturing x items will give a weekly cost given by $C(x)$.
$C(x) = 0.0005x^3 - 0.6x^2 + 250x + 700$

A) Find the cost of producing the 201^{st} item.

B) Find the cost of the 343^{rd} item.

C) Compute the marginal cost at a production level of 200 items per week and interpret the results.

D) Compute the marginal cost at a production level of 342 items per week and interpret the results.

E) If the weekly production is 342 items and we are going to increase the weekly production to 345 items, what is the approximate change to the weekly cost?

F) Estimate the cost of the 250^{th} item.

Example: The revenue function for a business is given by $R(x)$, in dollars, and x is in hundreds of items.

$R(x) = 0.02x^4 + 0.5x^2$

A) Compute and interpret $R'(5)$.

B) Estimate the revenue for selling the 301^{st}item thru the 400^{th} item.

Definition: Suppose that x is the number of units of a product produced. These computations give the cost, revenue, or profit per unit of production

average cost $= \overline{C}(x) = \dfrac{C(x)}{x}$ **average revenue** $= \overline{R}(x) = \dfrac{R(x)}{x}$

average profit $= \overline{P}(x) = \dfrac{P(x)}{x} = \dfrac{R(x) - C(x)}{(x)} = \dfrac{R(x)}{x} - \dfrac{C(x)}{x} = \overline{R}(x) - \overline{C}(x)$

Definition: The notation for the marginal average cost, marginal average revenue, and marginal average profit functions is

marginal average cost $= \overline{C}'(x)$

marginal average revenue $= \overline{R}'(x)$

marginal average profit $= \overline{P}'(x)$

Example: A company knows that manufacturing x items will give a weekly cost given by $C(x)$.
$C(x) = 0.0005x^3 - 0.6x^2 + 250x + 700$

A) Compute and interpret $C(200)$, $\overline{C}(200)$

B) Find the average cost for a weekly production of 300 items.

C) Find the marginal average cost function.

D) Compute and interpret $\overline{C}'(200)$

E) Estimate the average cost of producing 201 items.

F) Compute and interpret $\overline{C}'(300)$

Example: Suppose that $R(x)$ is the revenue, in dollars, for selling x items.

$R(x) = -0.04x^2 + 660x$

A) Compute and interpret $\overline{R}(500)$

B) Compute $\overline{R}'(500)$

Example: At a weekly production level of 3000 items, the marginal cost is \$60/item and the marginal revenue is \$83/item. What can be said about the profit of the 3001^{st} item?