Example: Find the derivative of $y = (x^2 + 5)^{20}$

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Chain Rule

If $y = f(u)$ and $u = g(x)$ define the function

$y = f(g(x))$ then

$$\frac{dy}{dx} =$$

General Power Rule

$y = [f(x)]^n$

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Example: Find the derivatives of these functions. Do not simplify.

A) $y = (x^3 + 2x + 6)^5$

B) $y = \sqrt{x^5 + 6}$

C) $y = (x^2 + 5)^3 (x^5 + 7)^4$

D) $y = \dfrac{x^4 + 7}{(x^5 + 1)^3}$

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## Exponential Rules

$y = e^{f(x)}$

$y = a^{f(x)}$

Example: Find the derivatives of these functions. Do not simplify.

A) $y = e^{x^4}$

B) $y = 5^{x^3 + 1}$

C) $y = x^2 + e^{3x}$

D) $y = x^2 + 3^{5x}$

Example: Find the values of $x$ where the tangent line is horizontal.

$y = \dfrac{e^{6x} e^{x^2}}{e^{3x-5}}$

Logarithm Rules

$y = \ln(f(x))$

$y = \log_b(f(x))$

Example: Find the derivatives of these functions. Do not simplify.

A) $y = \ln(x^2 + 5)$

B) $y = \log_3(x^5 + 5x + 1)$

C) $y = \ln(20x^4)$

D) $y = \left[\log_4(x^2)\right]^5$

E) $y = \ln(\sqrt{x^5 + 3})$

F) $y = \ln[(x^3 + 7)^4(x^2 + 4)^5]$

Combining Rules

Example: Find the derivatives of these functions. Do not simplify after taking the derivative.

A) $y = \dfrac{x^4}{7^{2x}}$

B) $y = \left(\dfrac{x^4 + 3}{x^3 + 1}\right)^9$

C) $y = \sqrt[3]{x^2 + \sqrt{2x}}$

Example: Suppose that $C(50) = \$5$ and $C'(50) = \$5$/item.

A) Compute $\overline{C}\,'(50)$.

B) Estimate the average cost of 51 items.

Example: Find the x-values where the slope of the tangent line is zero.

$y = (x+1)^3(x-4)^4$

Example: Find the x-values where the slope of the tangent line is zero.

$$y = \frac{x^2 - 25}{x^2 - 9}$$

Example: If you know that $g(5) = -4$ and $g'(5) = 3$, compute $H'(2)$.
$H(x) = x^3 * g(x^2 + 1)$