Section 5.2 and 5.4: Second Derivatives and Curve Sketching

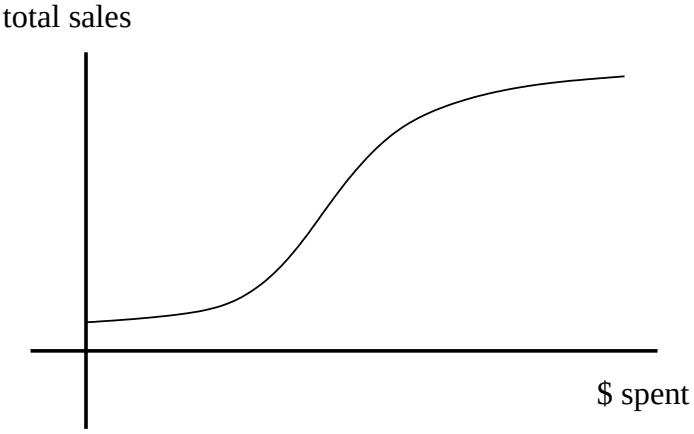
Higher Order Derivatives

Example: Find the indicated derivatives.
A) $y = x^4 + 3x^2 + 7x + 1$, y', y'', y''', $y^{(4)}$

B) $y = 36x^{45} + 3x^{40} + 8x^7 + 6$, $y^{(47)}$

C) $y = x^3 e^{3x}$, y''

The following is a graph that gives the total sales of a product verses the amount of money spent on advertising.



$f(x)$ is concave up $\leftrightarrow$

$f(x)$ is concave down $\leftrightarrow$

Definition: $x = a$ is called a **possible inflection value** of $f(x)$ provided that

1)

2)

$f(x)$ will have an **inflection point** at $x = a$ provided the function changes concavity at $x = a$.

Example: Find the intervals of concavity. Does the function have an inflection point? If yes, give the location(s).

$y = -2x^3 + 3x^2 + 12x$

Example: Find the intervals of concavity. Does the function have an inflection point? If yes, give the location(s).

$y = x^5 - 10x^4 + 6x + 5$

Example: Sketch a graph that has the following properties.

$\lim_{x \to \infty} f(x) = 4$

$f(x)$ is continuous

$f'(x) > 0$ on $(-\infty, 0)$

$f'(x) < 0$ on $(0, \infty)$

$f''(x) > 0$ on $(-\infty, 0)$

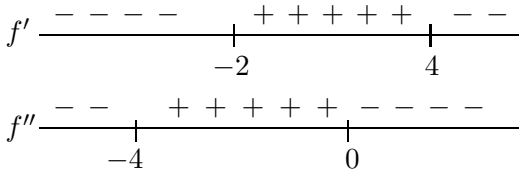
$f''(x) < 0$ on $(0, \infty)$

Example: Use the graphing strategy on this function. (See page 321)

$y = x + \dfrac{16}{x}$

Example: Sketch a graph that has the following properties.

$f(x)$ is continuous
$\lim_{x \to -\infty} f(x) = 0$



Example: Use the graphing strategy on this function.

$y = x^{2/3}$

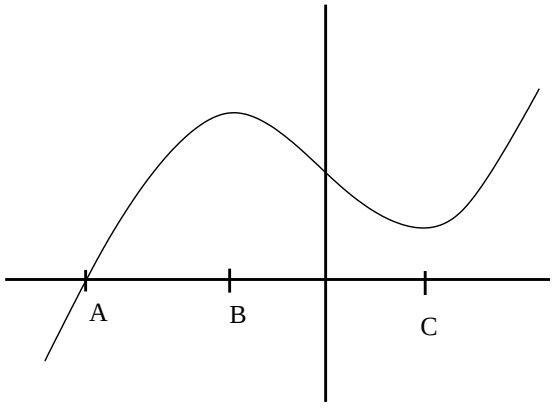
Example: Use the graphing strategy on this function.

$y = \dfrac{x}{(x-4)^2}$

Example: Find the values of A and B so that the function, $f(x)$, will have an inflection point at $x = 3$ and an instantaneous rate of change of 10 at $x = 1$.

$f(x) = Ax^3 - 18x^2 + Bx + 7$

Example: Use the graph of $f'(x)$ to answer these questions.



Give the intervals where is $f(x)$ increasing?

Give the intervals where is $f(x)$ concave up?

Give the intervals where is $f(x)$ concave down?