Section 6.4: Definite Integral

Example: The following data gives the speed of a car x seconds after the car starts to stop. Estimate the distance the car travels during this time period.

x(seconds)	0	2	4	6	8
speed(ft/sec)	50	40	25	10	0

The marginal revenue, in millions of dollars per year, for a product is given in the table.

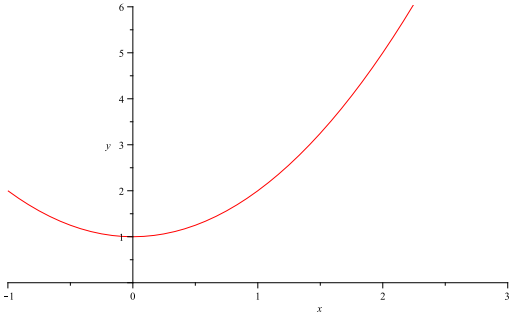
year	1990	1991	1992	1993	1994
rate	3.3	2.2	2.8	2.9	2.4

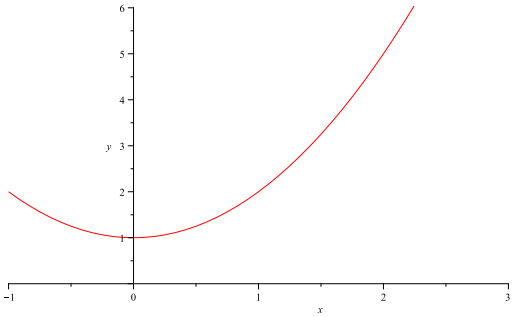
Approximate the revenue for this product from 1990-1993.

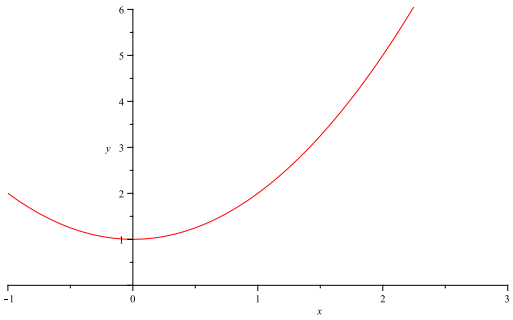
Riemann Sums

Terminology:
n = the number of rectangles
base of each rectangle $= \dfrac{b-a}{n}$
L_n is a left sum with n rectangles
R_n is a right sum with n rectangles
M_n is a midpoint sum with n rectangles

Example: Use the function $f(x) = x^2 + 1$ on the interval $[0, 2]$ to answer the following.







Definition: Let $f(x)$ be a continuous function on $[a, b]$. The **definite integral** of f from a to b is defined and denoted in the following manner.

The **integrand** is $f(x)$, the **lower limit of the integral** is a, and the **upper limit of the integral** is b.

Example: Estimate $\int_1^5 \ln(1+x^2)\ dx$ using 4 rectangles.

Properties of the definite integral

$$\int_a^a f(x)\ dx = 0$$

$$\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$$

$$\int_a^b f(x)\ dx + \int_b^c f(x)\ dx = \int_a^c f(x)\ dx$$

$$\int_a^b f(x) + g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx$$

$$\int_a^b k * f(x)\ dx = k\int_a^b f(x)\ dx$$

Example: Use the fact that $\int_a^b g(x)\ dx = 7$ and $\int_a^b f(x)\ dx = 4$ to compute $\int_a^b 5f(x) - 2g(x)\ dx =$

Interpretations of the definite integral

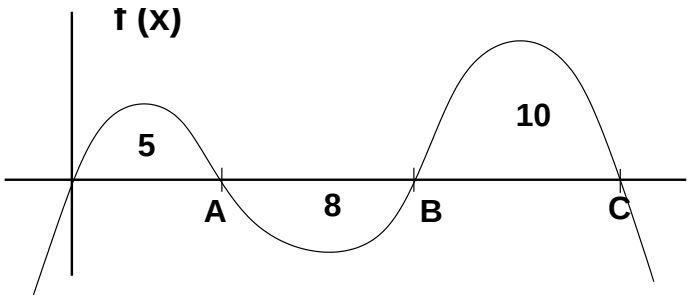
If $f(x) \geq 0$ on the interval $[a, b]$ then $\int_a^b f(x)\ dx$

will represent _____

If $f(x)$ is not entirely above the x-axis on the interval $[a, b]$

then $\int_a^b f(x)\ dx$ will represent _____

Example: Use the graph of $f(x)$ to answer these questions.



$\int_0^A f(x)\ dx =$ _____ $\int_A^B f(x)\ dx =$ _____ $\int_C^B f(x)\ dx =$ _____

$\int_A^C f(x)\ dx =$ _____ $\int_0^C f(x)\ dx =$ _____

Find the area between $f(x)$ and the x-axis from $x = A$ to $x = C$.