

Section 6.5: The Fundamental Theorem of Calculus

Fundamental Theorem of Calculus: If $f(x)$ is continuous on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$ then

$$\int_a^b f(x) dx = F(b) - F(a)$$

Example: Evaluate these definite integrals.

$$\text{A) } \int_1^4 \frac{2}{x} dx =$$

$$\text{B) } \int_1^3 3x^2 - 12 dx =$$

$$\text{C) } \int_1^3 4x(x^2 - 1)^3 dx =$$

$$\text{D) } \int_0^A 15e^{3x} dx =$$

Interpretation of a definite integral

If you have the rate of change of a function, $f'(x)$, then $\int_a^b f'(x) dx$ is the total change of $f(x)$ from $x = a$ to $x = b$ where $f(x)$ is an antiderivative of $f'(x)$.

Example: $A(t) = \frac{8t}{t^2 + 1}$ is the rate that water is pumped from a well given in gallons/min.

A) Interpret and compute $\int_0^5 A(t) dt$.

B) Find the total number of gallons pumped from the well during the second minute.

C) How long will it take to pump 15 gallons from the well?

Example: The monthly sales rate in items/month is given by

$$s(x) = \begin{cases} 18x^2 - 135x + 2882 & \text{if } 0 \leq x \leq 20 \\ 9x^2 - 816x + 20102 & \text{if } 20 < x \leq 30 \end{cases}$$

Find the total number items sold in the first 27 months the product was on the market.

Definition: The **average value** of a function $f(x)$ on the interval $[a, b]$ is given by

Avg =

Example: Bob deposits \$4000 into an account earning 6% interest compounded continuously. Determine the average balance of the account during the first three years.

Example: The profit function for making and selling x item is $P(x) = -0.01x^2 + 55x - 800$. Find the average profit from $x = 2000$ to $x = 6000$.

Example: The total accumulated cost, $C(x)$ and revenue, $R(x)$, in hundreds of dollars for a video game satisfy

$$C'(x) = 3 \quad \text{and} \quad R'(x) = 15e^{-0.1x}$$

where x is the number of years the video game has been in service. Find the total profit accumulated during the useful life of the game.