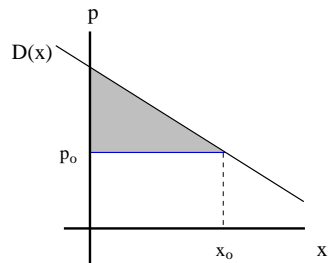


Section 7.2 (and 7.1): Applications of Area between Curves

Consumer and Producer Surplus

Definition: **Consumer surplus** is the total amount saved by consumers who were willing to pay a price higher than p_o but instead bought the item(s) at p_o .

This concept can be demonstrated on with the graph.



Example: The demand function, in dollars, for a product is given by $p = D(x) = -x^2 - x + 650$, where x is the number of items. Find the consumer surplus when the price is \$560.

Example: The demand function, in dollars, for a product is given by $p = D(x) = 320e^{-0.01x}$, where x is the number of items. Find the consumer surplus when the demand is 100 items.

Definition: **Producer surplus** is the total increase in money realized by producers who were willing to sell at a price lower than p_o but instead sold the item(s) at p_o .

Example: The supply function, in dollars, for a product is given by $p = S(x) = 0.2x^2 + 15$, where x is the number of items. Find the producer surplus when $x = 110$.

Example: Find the producer and consumer surplus at market equilibrium for

$$D(x) = 2528 - 0.01x^2$$

$$S(x) = 0.02x^2 + 5$$

Continuous Income Streams

Example: The rate of flow of income produced by a vending machine is $f(x) = 3000e^{0.057x}$, where x is in years and $0 \leq x \leq 5$ and $f(x)$ is the flow in dollars/year.

What is the total income produce by the vending machine in these 5 years?

Suppose that we have an income stream $f(x)$ and this income stream is invested at $r\%$ continuous interest over T years. What is the future value of this income stream.

Example: Find the future value of the income stream provided by the vending machine if the income is invested at 4.5% compounded continuously.

Example: I am willing to sell this investment(now) for \$16,000. Would this be a good investment to buy?

Definition: The **present value** of an investment is the amount of money invested now at $r\%$ that will give the future value of the investment after T years.

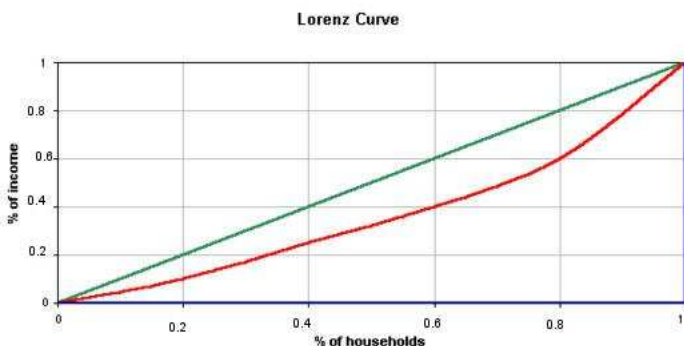
Example: The continuous income stream of an investment is $f(x) = x^3 - 8x^2 + 250$, where x is in months and $f(x)$ is in dollars per month. Assuming that current interest rate is 9% compounded continuously, find the future value and the present value of this investment if it is held 1.5 years.

Lorenz curve and Gini Index

The Lorenz curve, $y = f(x)$, is a function that represents income distribution in a society. Sometimes called a graphical representation of the cumulative distribution function of a probability distribution.

x represents the cumulative percentage of families at or below a given income level.

y represents the cumulative percentage of total family income received.



Example: Read the value of $f(0.6)$ from the graph and explain its meaning.

Definition: The **Gini index**, gini coefficient, is a measure of the income distribution.

$$\text{Gini index} = 2 \int_0^1 x - f(x) \, dx$$

Example: The Lorenz curve for a country is given by $f(x) = x^{2.2}$. Calculate the Gini index.

Example: A study was conducted and it found that the distribution of income for stockbrokers, given by $f(x)$, and the distribution of income for teachers, given by $g(x)$, was the following

$$f(x) = \frac{11x^2}{12} + \frac{x}{12} \qquad g(x) = \frac{5x^2}{6} + \frac{x}{6}$$

Determine which profession has the more equitable distribution of income.