Appendix K.2: Slopes and Tangents of Parametric Curves

Suppose that a curve, C, is described by the parametric equations $x = x(t)$ and $y = y(t)$ or the vector function $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ where both $x(t)$ and $y(t)$ are differentiable. Then the slope of the tangent line is given by

$$\text{slope} = \frac{y'(t)}{x'(t)}.$$

$$\frac{dy}{dx} =$$

Example: Find $\dfrac{dy}{dx}$ and $\left.\dfrac{dy}{dx}\right|_{t=3}$ and $\left.\dfrac{dy}{dx}\right|_{(5,-1)}$

$x(t) = t^3 - 3t^2 + 5$
$y(t) = 2t - 7$

Example: Find the equation of the tangent line at $t = 0$.

$x(t) = e^{t^2+4t}$
$y(t) = 5^{3t+2}$

Example: Compute the derivatives at the point $(0, 0)$.
$x(t) = \sin(2t)$
$y(t) = \cos(t)$

Horizontal tangent lines

Vertical tangent lines

Example: Find the points on the curve where the tangent lines are horizontal and where they are vertical.

$x = t^2 + t$
$y = t^2 - t$

Example: Find the values of t where the tangent lines are horizontal and where they are vertical.

$x = t + 3$
$y = t^3 - 3t^2$