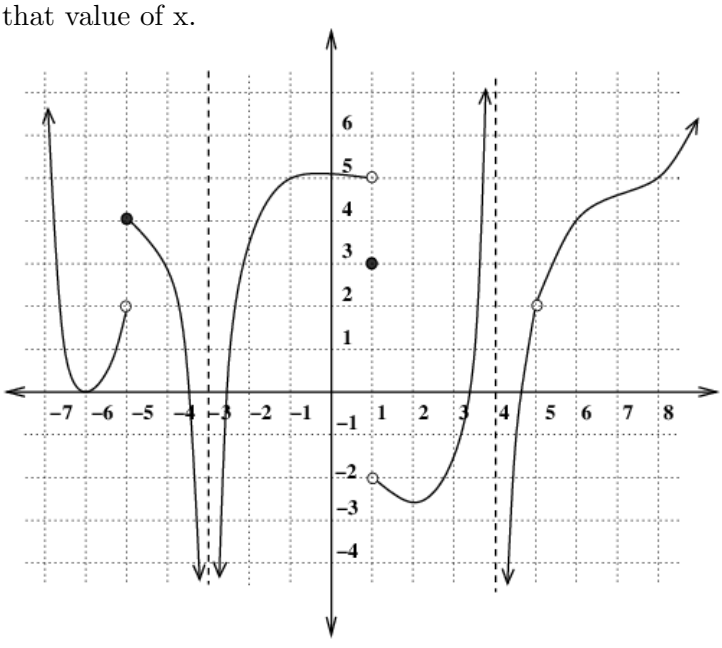
Section 2.5: Continuity

Definition: A function f is **continuous at a number** $x = a$ if $\lim_{x \to a} f(x) = f(a)$

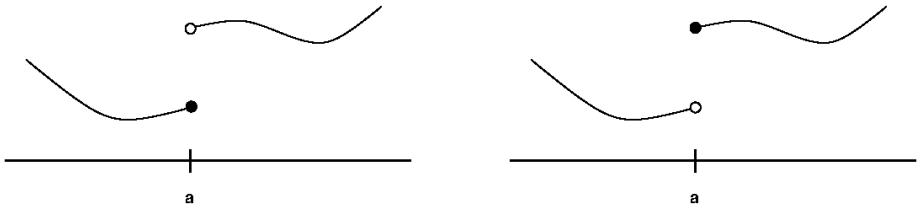
Example: Is the function $f(x) = x^2 + 1$ continuous at $a = 3$, i.e. at $x = 3$?

Example: Where is the function $f(x)$ discontinuous? Explain what type of discontinuity happens at that value of x.



Definition: A function f is **continuous from the right at a number** a if $\lim_{x \to a^+} f(x) = f(a)$.

A function f is **continuous from the left at a number** a if $\lim_{x \to a^-} f(x) = f(a)$.



A function f is continuous on an interval if it is continuous at every number in the interval. At the endpoint of the interval we understand continuous to mean left or right continuity.

Example: Discuss the continuity of the function $f(x) = \dfrac{x+5}{x-4}$.

Example: Discuss the continuity of the function $f(x) = \dfrac{x^2+x-2}{x-1}$.

Example: What would you define $f(1)$ to make the function continuous? i.e. Find the value of A so that $f(x)$ is continuous.

$$f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & \text{if } x \neq 1 \\ A & \text{if } x = 1 \end{cases}$$

Example: Find the values where $f(x)$ is not continuous. Then classify the value(s) as a vertical asymptote or removable discontinuity.

$$f(x) = \frac{x^2+2x}{x^4-3x^3-10x^2}$$

Example: Find the value(s) where $f(x)$ is not continuous.

$$f(x) = \begin{cases} 3x+1 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$$

Example: Find the value(s) where $f(x)$ is not continuous.

$$f(x) = \begin{cases} 3x & \text{if } x < 2 \\ x+4 & \text{if } x > 2 \end{cases}$$

Example: Find the value(s) of A that will make $g(x)$ a continuous function.

$$g(x) = \begin{cases} A^2x & \text{if } x \leq 1 \\ 3Ax-2 & \text{if } x > 1 \end{cases}$$

Example: Find the value(s) where $f(x)$ is not continuous.

$$f(x) = \begin{cases} 3x^2 + 4x + 1 & \text{if } x \leq 2 \\ \dfrac{5x^2 + 1}{x - 1} & \text{if } x > 2 \end{cases}$$

Intermediate Value Theorem: Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number such that N is strictly between $f(a)$ and $f(b)$. There there exist a number c with $a < c < b$ such that $f(c) = N$.

Example: Use the Intermediate Value Theorem to show that there is areal number a such that $f(a) = 12$.

$f(x) = -x^4 + 3x^3 + 5$

Example: Show that $f(x) = x^4 - 5x^2$ and $g(x) = 2x^3 - 4x + 6$ intersect between $x = 3$ and $x = 4$.

Example: A student did the following work on a question on an exam. The student showed that $f(1) = 1$ and $f(-1) = -1$ for the given function and then claimed by the Intermediate Value Theorem that there was some number c with $-1 < c < 1$ such that $f(c) = 0$. Did the student receive full credit on this problem?