Section 2.6: Limits at Infinity

The end behavior of a function is computed by $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$. If either of these limits is a number, L, then $y = L$ is called a **horizontal asymptote** of $f(x)$.

Example: Compute these limits.

A) $\lim\limits_{x\to\infty} \arctan(x) =$

B) $\lim\limits_{x\to-\infty} \arctan(x) =$

C) $\lim\limits_{x\to\infty} \left(x^2 - 4x + 2\right) =$

D) $\lim\limits_{x\to\infty} \left(x^2 - x^5\right) =$

E) $\lim\limits_{x\to\infty} \left[3 - 2\left(\frac{\pi}{4}\right)^x\right] =$

F) $\lim\limits_{x\to-\infty} \left[3 - 2\left(\frac{\pi}{4}\right)^x\right] =$

G) $\lim\limits_{x\to\infty} \dfrac{80}{4 + 2e^{-0.15x}} =$

H) $\lim\limits_{x\to-\infty} \dfrac{80}{4 + 2e^{-0.15x}} =$

Theorem: If $r > 0$ is a rational number such that x^r is defined for all x, then

$\lim_{x\to\infty} \frac{1}{x^r} = 0$ and $\lim_{x\to-\infty} \frac{1}{x^r} = 0$

Example: Compute these limits.

A) $\lim_{x\to\infty} \frac{3x^2+1}{2x^3+x^2} =$

B) $\lim_{x\to-\infty} \frac{3x^4+7}{x+2} =$

C) $\lim_{x\to\infty} \frac{\sqrt{2x^2+3}}{x+2} =$

D) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{2x^2+3}}{x+2} =$

E) $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2+3x} - x\right)$

F) $\lim_{x\to-\infty} \frac{\sqrt{3x^4+1}}{2x^2+7} =$

G) $\lim_{x\to\infty} \frac{2e^{3x}+e^{-2x}}{3e^{4x}+5e^{-2x}}$

H) $\lim_{x \to -\infty} \frac{2e^{3x} + e^{-2x}}{3e^{4x} + 5e^{-2x}}$

I) $\lim_{x \to \infty} \left[\ln(3x+5) \;\; - \ln(2+5x) \right]$

J) $\lim_{x\to\infty} \left[\ln(x+5) - \ln(2+x^2)\right]$

Example: Find the horizontal asymptotes of these functions.

$f(x) = \dfrac{x^4+3}{7x^5+8}$

$g(x) = \dfrac{x^4+3x^5}{7x^5+8}$