Section 3.1: Derivatives of Polynomials and Exponential Functions

Theorem: If f is a constant function, $f(x) = c$, then $f'(x) = 0$.

Theorem: If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$

Theorem: Let c be a constant and let $f'(x)$ and $g'(x)$ exists, then

a) if $y = cf(x)$, then $y' = cf'(x)$

b) if $y = f(x) \pm g(x)$, then $y' = f'(x) \pm g'(x)$

Example: Find the derivatives of these functions.

A) $y = 5, \quad y = \sqrt{8}, \quad y = \pi^4, \quad y = \sin(20^o)$

B) $y = x^{10}$

C) $y = 3x^5$

D) $B(x) = 3 - 7x + 4x^5$

Examine the derivative of $f(x) = a^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x a^h - a^x}{h} = \lim_{h\to 0} \frac{a^x(a^h - 1)}{h} = a^x \lim_{h\to 0} \frac{(a^h - 1)}{h}$$

For $a = 2$ then $\lim_{h\to 0} \frac{(2^h - 1)}{h} = 0.69$ and for $a = 3$ then $\lim_{h\to 0} \frac{(3^h - 1)}{h} = 1.10$.

Thus by the Intermediate value theorem, there is a number between 2 and 3 such that $\lim_{h\to 0} \frac{(a^h - 1)}{h} = 1$. This number is $e = 2.71828.....$

Example: Find the indicated derivative of these functions.

A) y' if $y = 3e^x + 2x^e$

B) y'' if $y = \sqrt[3]{x^8} + \sqrt{x^5} + e^{x+4}$

C) $f'''(x)$ if $f(x) = 3x^6 + 2x + 5$

D) y' if $y = 3a^{-5} + \dfrac{1}{2a^3} + 3^8$

E) y' if $y = \dfrac{m^3 + 5m^2 + 7}{m}$

F) y' if $y = \dfrac{x^4 + 1}{x^2\sqrt{x}}$

Example: Find the equation of the tangent line and the normal line to $f(x) = x^2 + 5x + 10$ at $x = 3$

Example: Find the value(s) of x where $f(x)$ has a tangent line that is parallel to $y = 6x + 5$

$f(x) = x^3 - 5x^2 + 6x - 30$

Example: Find the equation of the line(s) thru the point $(-1, -3)$ that are tangent to $y = x^2 + 7x + 12$

Example: Find $g'(x)$ when $g(x) = \begin{cases} 1-2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ x & \text{if } x > 1 \end{cases}$

Example: Find $k'(x)$ when $k(x) = \begin{cases} 4x^2 + 2x + 4 & \text{if } x < 1 \\ 10x - 3 & \text{if } x \geq 1 \end{cases}$

Example: Find $k'(x)$ when $k(x) = \begin{cases} 4x^2 + 2x + 1 & \text{if } x < 1 \\ 10x - 3 & \text{if } x \geq 1 \end{cases}$