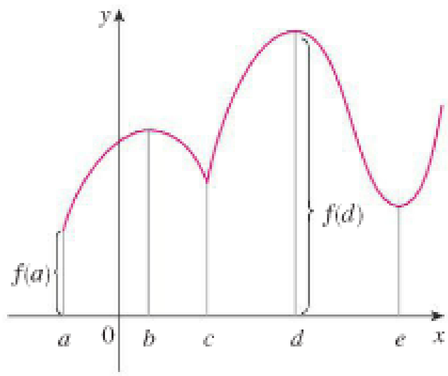
Sections 4.1-4.3 Part 3: Absolute Maximum/Minimum and other Theorems

Absolute Maxima and Minima

Definition: Let c be a number in the domain of a function f. Then $f(c)$ is the

- **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in the domain.
- **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in the domain.



Abs min $f(a)$
abs max $f(d)$
loc min $f(c), f(e)$
loc max $f(b), f(d)$

Example: Find the absolute max and the absolute min.

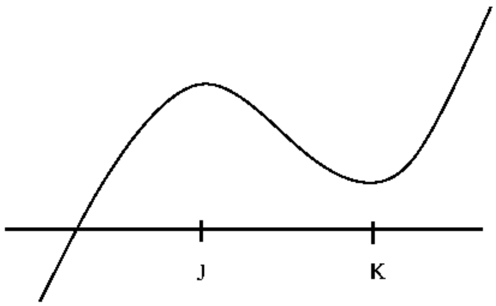
A) $y = x^3 + 3x^2 + 1$

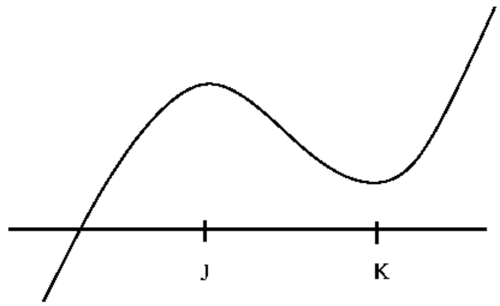
B) $y = x^4 - 4x^3$

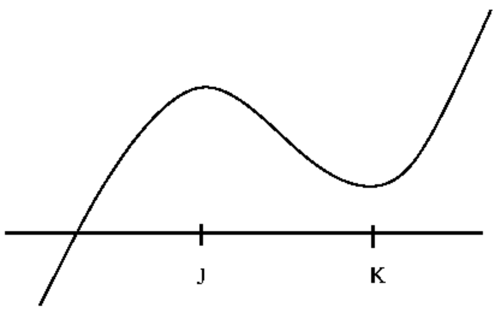
C) $y = 7 + 3\sin(x + 10)$

The Extreme Value Theorem: If f is a continuous on a closed interval $[a, b]$, then f will have both an absolute max and an absolute min. They will happen at either critical values in the interval or at the ends of the interval, $x = a$ or $x = b$.

Restricted Domains:







Example: For the function, find the absolute max and the absolute min on the indicated interval.

$f(x) = 12x^2 - 2x^3 + 1$ $\qquad$ $f'(x) = 24x - 6x^2 = 6x(4 - x)$

A) $[2, 5]$

B) $[-3, 5]$

C) $(-3, 5]$

Example: For the function, find the absolute max and the absolute min on the interval $[0, 5]$.

$$f(x) = \frac{1}{(x-4)^2}$$

Rolle's Theorem: Let f be a function that satisfies the following three hypotheses:

1) f is continuous on the closed interval $[a, b]$.
2) f is differentiable on the open interval (a, b).
3) $f(a) = f(b)$

Then there is a number c between a and b such that $f'(c) = 0$.

The Mean Value Theorem: Let f be a function that satisfies the following hypotheses:

1) f is continuous on the closed interval $[a, b]$.
2) f is differentiable on the open interval (a, b).

Then there is a number c between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example: Find a number c that satisfies the conclusion of the Mean Value Theorem on the interval $[0, 2]$.

$f(x) = x^3 + x - 1$

Example: You enter a toll road at 8am and then exit it at 9:15am. The distance between the entrance and exit is 100 miles. If the maximum speed is set at 70mph, do you get charged for speeding?