Sections 4.9: Antiderivatives

Definition: A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example: Is the function $F = x\ln(x) - x$ an antiderivative of $f = \ln(x)$?

Example: Find an antiderivative of $f = 2x$.

Theorem: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$, where C is an arbitrary constant.

Table of Antidifferentiation Formulas

Function	Antiderivative	Function	Antiderivative
$cf(x)$		$\sin(x)$	
$f(x) \pm g(x)$		$\cos(x)$	
x^n, if $n \neq -1$		$\sec^2(x)$	
x^{-1}		$\sec(x)\tan(x)$	
e^{kx}		$\frac{1}{x^2+1}$	
b^x		$\frac{1}{\sqrt{1-x^2}}$	

Example: Find the most general antiderivative.

A) $f(x) = 7x^4 + 3x^2 + 7$

B) $f(x) = \sqrt{x} + \sqrt[3]{x^5} + 3^4$

Example: Find $f(x)$

A) $f'(x) = x^2(x^5 + 2x)$

B) $f'(x) = e^{4x} + \sec(x)\tan(x) + 3^x$

C) $f'(x) = \dfrac{3}{x^4} + \dfrac{1}{5x^3} + \dfrac{4}{x} + \dfrac{1}{e^{3x}} + \dfrac{5}{7^{-x}}$

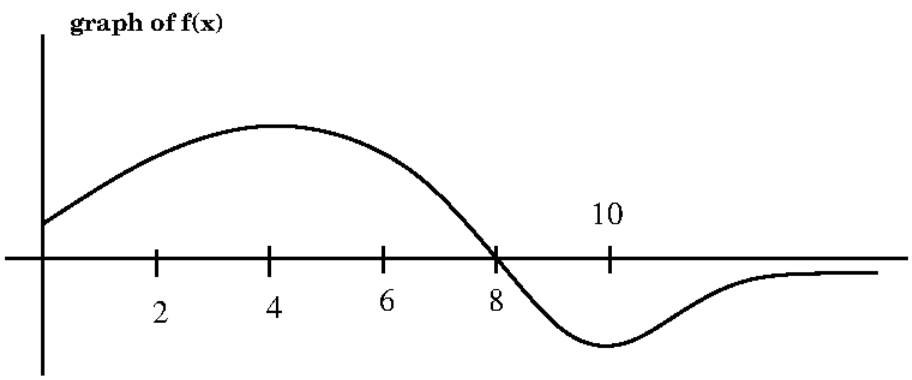
D) $f'(x) = \dfrac{x^3 + 2x + 7}{x^2}$

E) $f'(x) = -2(1 + x^2)^{-1}$

Definition: The vector function $R(t) = X(t)\mathbf{i} + Y(t)\mathbf{j}$ is an antiderivative $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$ if $R'(t) = r(t)$.

Example: Find the most general antiderivative of $r(t) = (3t^2 + 2)\mathbf{i} + (\sec^2(t))\mathbf{j}$.

Example: The graph of the function f is given in the figure. Make a rough sketch of the antiderivative F, given that $F(0) = -5$.



Example: Find $f(x)$ if $f'(x) = 3x^2 + 15e^{3x} + 4$ given $f(0) = 7$.

Example: Find $f(x)$ if $f''(x) = 20x^3 + 3\sin(x)$ and $f(0) = 2$ and $f'(0) = 8$.

Example A ball is thrown upward with a velocity of 50ft/sec from the edge of a 150 foot tall building.

A) Find a formula that gives the height of the ball after x seconds.
B) When does the ball reach its maximum height?
C) How fast does the ball hit the ground?

Example: A car braked with a constant deceleration of 50ft/sec^2, producing skid marks measuring 160ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

Example: A model rocket is launched from the ground. For the first two seconds, the rocket has an acceleration of $a(t) = 12t\ m/sec^2$. At this time all its fuel is spent and it becomes freely falling body.

A) Determine the position function and the velocity function for all times.
B) At what time does the rocket reach its maximum height, and what is that height?